AN OVERVIEW OF VARIATIONAL ANALYSIS

Lectures aimed at introducing the basic themes of variational analysis to researchers who are interested in optimization and who want a level of mathematical understanding and capability beyond convex analysis

1. Origins and Motivations

Why and how variational analysis developed from historical subjects such as nonlinear programming and the calculus of variations, while departing in major ways from the framework of classical analysis.

2. Variational Geometry

The tangent and normal subpaces associated with classical geometry of curves and surfaces need to be replaced by one-sided cones of tangent vectors and normal vectors, each of two different kinds. Polarity relations underlie a concept of variational regularity.

3. Subgradients and Optimality

The subgradients of convex analysis can be extended with localizing adjustments to nonconvex analysis in a manner compatible with traditional gradients. This amounts to applying variational geometry to epigraphs. It leads to a nonsmooth calculus that supports optimality conditions.

4. Variational Approximation

When can one problem of optimization be said to be a close approximation of another? This is a key question for which the answer should imply closeness of optimal values and solutions, but classical concepts of analysis miss the target.

5. Solution Mappings and Stability

Whan a problem is described by an equation with parameters, the standard implicit function theorem describes how the solution depends on those parameters. Problems of optimization are not modeled simply by equations, so more is needed for understanding such dependence, which relates to error analysis.