Math 308 **Final** **December 2011**

- We will write $\underline{A}_1, \dots, \underline{A}_n$ for the columns of an $m \times n$ matrix A.
- Several questions involve an unknown vector $\underline{x} \in \mathbb{R}^n$. We will write $x_1, \dots, x_n$ for the entries of $\underline{x}$; thus $\underline{x} = (x_1, \dots, x_n)^T$.
- The null space and range of a matrix A are denoted by $\mathcal{N}(A)$ and $\mathcal{R}(A)$, respectively.
- The linear span of a set of vectors is denoted by $\mathrm{Sp}(\underline{v}_1, \dots, \underline{v}_n)$.
- We will write $\underline{e}_1, \dots, \underline{e}_n$ for the standard basis for $\mathbb{R}^n$. Thus $\underline{e}_i$ has a 1 in the i^{th} position and zeroes elsewhere.
- In order to save space I will often write elements of $\mathbb{R}^n$ as row vectors, particularly in questions about linear transformations. For example, I will write $T(x, y) = (x + y, x - y)$ rather than

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x-y \end{pmatrix}.$$

Part A.

True or False.

Scoring. You get +1 for each correct answer, −1 for each incorrect answer and 0 if you choose not to answer the question. Use your BUBBLES: A=True. B=False. Fill in bubble A if you think it is True, bubble B if you think it is False, and fill in nothing if you do not want to answer it.

(1) If a is a non-zero real number, the matrix $\begin{pmatrix} a & a \\ -a & 0 \end{pmatrix}$ has no real eigenvalues.
(2) Every set of five vectors in $\mathbb{R}^4$ is linearly dependent.
(3) Every set of four vectors in $\mathbb{R}^4$ is linearly dependent.
(4) Every set of five vectors in $\mathbb{R}^4$ spans $\mathbb{R}^4$.
(5) Every set of four vectors in $\mathbb{R}^4$ spans $\mathbb{R}^4$.
(6) A square matrix is non-singular if all its entries are non-zero.
(7) A square matrix having a row of zeroes is always singular.
(8) For any vectors $\underline{u}$, $\underline{v}$, and $\underline{w}$, $\{\underline{u}, \underline{v}, \underline{w}\}$ and $\{\underline{u}+\underline{v}, \underline{v}, \underline{w}+\underline{u}\}$ have the same linear span.
(9) If A is singular and B is non-singular then AB is always singular.
(10) If A and B are non-singular so is AB.
(11) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix}$ have the same the linear span as $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$.
(12) There is a matrix whose inverse is $\begin{pmatrix} 3 & 4 & 5 \\ 2 & 5 & 7 \\ 5 & 9 & 12 \end{pmatrix}$.
(13) If $A^{-1} = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 0 & 2 \\ 1 & 0 & 1 \end{pmatrix}$ and $E = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \end{pmatrix}$ there is a matrix B such that $BA = E$.

(14) If A is row-equivalent to the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$, then the equation $A\underline{x} = \underline{b}$ has a unique solution.

(15) The matrix representing the linear transformation $T(x, y) = (x + y, x - 2y)$ is $\begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}$.

(16) Let S and T be the linear transformation $T(x, y) = (2x + y, x - y)$ and $S(x, y) = (y, 2y)$. Then $ST(x, y) = (x - y, 2x - 2y)$.

(17) There exists a 3×4 matrix A and a 4×3 matrix B such that AB is the 3×3 identity matrix.

(18) There exists a 4×3 matrix A and a 3×4 matrix B such that AB is the 4×4 identity matrix.

(19) The dimension of a subspace is the number of elements in it.

(20) Every subset of a linearly dependent set is linearly dependent.

(21) Every subset of a linearly independent set is linearly independent.

(22) If $\{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$ are any vectors in $\mathbb{R}^n$, then $\{\underline{v}_1 + 3\underline{v}_2, 3\underline{v}_2 + \underline{v}_3, \underline{v}_3 - \underline{v}_1\}$ is linearly dependent.

(23) Let A and B be $n \times n$ matrices. Suppose 2 is an eigenvalue of A and 3 is an eigenvalue of B. Then 6 is an eigenvalue of AB.

(24) Let A and B be $n \times n$ matrices. Suppose 2 is an eigenvalue of A and 3 is an eigenvalue of B. Then 5 is an eigenvalue of $A + B$.

(25) Let A and B be $n \times n$ matrices. If $\underline{x}$ is an eigenvector for both A and B it is also an eigenvector for AB.

(26) Let A and B be $n \times n$ matrices. If $\underline{x}$ is an eigenvector for both A and B it is also an eigenvector for $3A - 2B$.

(27) If A is an invertible matrix, then $A^{-1}\underline{b}$ is a solution to the equation $A\underline{x} = \underline{b}$.

(28) The linear span $\mathrm{Sp}\{\underline{u}_1, \ldots, \underline{u}_r)$ is the same as the linear span $\mathrm{Sp}(\underline{v}_1, \ldots, \underline{v}_s)$ if and only if every $\underline{u}_i$ is a linear combination of the $\underline{v}_j$s and every $\underline{v}_j$ is a linear combination of the $\underline{u}_i$s.

(29) If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and V is a subspace of $\mathbb{R}^n$, then $T(V)$ is a subspace of $\mathbb{R}^m$.

(30) The row reduced echelon form of a square matrix is the identity if and only if the matrix is invertible.

(31) Let A be an $n \times n$ matrix. If the columns of A are linearly dependent, then A is singular.

(32) If A and B are $m \times n$ matrices such that B can be obtained from A by elementary row operations, then A can also be obtained from B by elementary row operations.

(33) There is a matrix whose inverse is $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$

(34) The column space of the matrix $\begin{pmatrix} 3 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 0 & 3 \end{pmatrix}$ is a basis for $\mathbb{R}^3$.

(35) Two systems of m linear equations in n unknowns have the same row reduced echelon form if and only if they have the same solutions.

(36) If $\underline{u}$ and $\underline{v}$ are $n \times 1$ column vectors then $\underline{u}^T\underline{v} = \underline{v}^T\underline{u}$.

(37) If $A^3 = B^3 = C^3 = I$, then $(ABAC)^{-1} = C^2A^2B^2A^2$.
(38) Let A be a non-singular 5×5 matrix and $\{\underline{u}_1, \underline{u}_2, \underline{u}_3\}$ a subset of $\mathbb{R}^5$. Then $\{A\underline{u}_1, A\underline{u}_2, A\underline{u}_3\}$ is linearly independent if and only if $\{\underline{u}_1, \underline{u}_2, \underline{u}_3\}$ is.
(39) If W is a subspace of $\mathbb{R}^n$ that contains $\underline{u} + \underline{v}$, then W contains $\underline{u}$ and $\underline{v}$.
(40) There is a 5×5 matrix having eigenvalues 1 and 2 and no others.
(41) There is a 5×5 matrix having eigenvalues $1, 2, 3, 4, 5$ and no others.
(42) There is a 5×5 matrix having eigenvalues $1, 2, 3, 4, 5, 6, 7$ and no others.
(43) A 5×5 matrix can't have more than 5 eigenvectors.
(44) A 5×5 matrix has exactly 5 eigenvalues.
(45) The vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is an eigenvector for the matrix $\begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix}$.
(46) The vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ is an eigenvector for the matrix $\begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix}$.
(47) If $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is an eigenvector for a matrix so is $\begin{pmatrix} 10 \\ 20 \end{pmatrix}$.
(48) If $\underline{u}$ and $\underline{v}$ are eigenvectors for A is is $\underline{u} + 2\underline{v}$.
(49) One way to compute the λ-eigenspace of a square matrix A is to compute the null space of $A - \lambda I$.
(50) If $\underline{u}$ and $\underline{v}$ are linearly independent vectors on the plane in $\mathbb{R}^4$ given by the equations $x_1 - x_2 + x_3 - 4x_4 = 0$ and $x_1 - x_2 + x_3 - 2x_4 = 0$, then $(1, 2, 1, 0)$ a linear combination of $\underline{u}$ and $\underline{v}$.
(51) The vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is an eigenvector for the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix}$.
(52) The vector $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$ is an eigenvector for the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix}$.
(53) The number 0 is an eigenvalue for the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix}$.
(54) The range of a matrix is its columns.
(55) The formula $T(a, b, c) = 0$ defines a linear transformation $\mathbb{R}^3 \to \mathbb{R}$.
(56) The formula $T(a, b, c) = 1$ defines a linear transformation $\mathbb{R}^3 \to \mathbb{R}$.
(57) Let $|a|$ denote the absolute value. The formula $T(a, b, c) = |a| + |b| + |c|$ defines a linear transformation $\mathbb{R}^3 \to \mathbb{R}$.
(58) The formula $T(a, b, c) = (a, b, 1)$ defines a linear transformation $\mathbb{R}^3 \to \mathbb{R}^3$.
(59) The vectors $(2, 2, -4, 3, 0)$ and $(0, 0, 0, 0, 1)$ are a basis for the subspace $x_1 - x_2 = 2x_2 + x_3 = 3x_1 - 2x_4 = 0$ of $\mathbb{R}^5$.
(60) The vectors $(1, 1)$, $(1, -2)$, $(2, -3)$ are a basis for the subspace of $\mathbb{R}^4$ give by the solutions to the equations $x_1 - x_2 = 2x_2 + x_3 = 3x_1 - 2x_4 = 0$.
(61) $\{\underline{x} \in \mathbb{R}^4 \mid x_1 - x_2 = x_3 + x_4\}$ is a subspace of $\mathbb{R}^4$.
(62) $\{\underline{x} \in \mathbb{R}^5 \mid x_1 - x_2 = x_3 + x_4 = 1\}$ is a subspace of $\mathbb{R}^5$.
(63) The solutions to a system of homogeneous linear equations form a subspace.
(64) The solutions to a system of linear equations form a subspace.
(65) The set $W = \{\underline{x} = (x_1, x_2, x_3, x_4)^T \in \mathbb{R}^4 \mid x_1^2 = x_2^2\}$ is a subspace.
(66) The null space of a square matrix A is equal to its 0-eigenspace.
(67) The linear span of a matrix is its set of columns.
(68) $U \cup V$ is a subspace if U and V are.

(69) U^{-1} is a subspace if U is.
(70) Similar matrices have the same eigenvalues.
(71) Similar matrices have the same eigenvectors.
(72) The equation $A\underline{x} = \underline{b}$ has a solution if and only if $\underline{b}$ is linear combination of the rows of A.
(73) The equation $A\underline{x} = \underline{b}$ has a unique solution for all $\underline{b} \in \mathbb{R}^n$ if A is an $n \times n$ matrix with rank n.
(74) If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation, then T is invertible if and only if its nullity is zero.
(75) A matrix is linearly independent if its columns are different.
(76) If A is a 3×5 matrix, then the inverse of A is a 5×3 matrix.
(77) The matrix $\begin{pmatrix} 18 & 6 \\ 10 & 1 \end{pmatrix}$ is diagonalizable.
(78) There is a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $\mathcal{R}(T) = \mathcal{N}(T)$.
(79) There is a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$ such that $\mathcal{R}(T) = \mathcal{N}(T)$.
(80) If A is a 2×2 matrix it is possible for $\mathcal{R}(A)$ to be the parabola $y = x^2$.
(81) Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation $T(x_1, x_2, x_3, x_4) = (0, x_1, x_2, x_3)$. The null space of T is $\{(0, 0, 0, a) \mid a \text{ is a real number}\}$.
(82) Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation $T(x_1, x_2, x_3, x_4) = (0, x_1, x_2, x_3)$. The null space of T is $\{(x_4, 0, 0, 0) \mid x_4 \text{ is a real number}\}$.
(83) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation $T(x_1, x_2) = (x_2, 0)$. The null space of T is $\{(t, 0) \mid t \text{ is a real number}\}$.
(84) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation $T(x_1, x_2) = (x_2, 0)$. The null space of T is $\{(1, 0)\}$.
(85) Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation $T(x_1, x_2, x_3) = (x_1, 0, x_2, x_2)$. The nullspace of T has many bases; one of them is the set $\{(-2, 0, 0, 0), (0, 0, 1, 1)\}$.
(86) Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation $T(x_1, x_2, x_3) = (x_1, 0, x_2, x_2)$. The nullspace of T has many bases; one of them is the set $\{(0, 0, 4)\}$.
(87) The set $\{(2, 0, 0, 0), (0, 0, 3, 3)\}$ is a basis for the range of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ given by $T(x_1, x_2, x_3) = (x_1, 0, x_2, x_2)$.
(88) The smallest subspace containing subspaces V and W is $V + W$.
(89) No linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^5$ is onto.
(90) No linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^3$ is onto.
(91) No linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^5$ is one-to-one.
(92) No linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^3$ is one-to-one.
(93) A linear transformation is invertible if and only if its nullity is zero.
(94) A linear transformation is one-to-one if and only if its nullity is zero.

In the next 6 questions, A is a 4×4 matrix whose columns $\underline{A}_1, \underline{A}_2, \underline{A}_3, \underline{A}_4$ have the property that $\underline{A}_1 + \underline{A}_2 + \underline{A}_3 = \underline{A}_4$.

(95) The columns of A span $\mathbb{R}^4$.
(96) A is singular.
(97) The columns of A are linearly dependent.
(98) The rows of A are linearly dependent.
(99) The equation $A\underline{x} = 0$ has a non-trivial solution.
(100) $A \begin{pmatrix} -2 \\ -2 \\ -2 \\ 2 \end{pmatrix} = 0$.

Part B.

Complete the definitions and theorems by completing the sentences.
Scoring: 2 points per question. No partial credit.

Systems of linear equations

(1) **Definition:** Two systems of linear equations are <u>equivalent</u> if _______________.
(2) **Theorem:** Two systems of linear equations are equivalent if their row reduced echelon forms are _______________.
(3) **Definition:** Let A be an $m \times n$ matrix and let E be the row-reduced echelon matrix that is row equivalent to it. If $x_1, \dots, x_n$ are the unknowns in the system of equations $A\underline{x} = \underline{b}$, then x_j is a dependent variable if and only if ________.
(4) **Theorem:** A homogeneous system of linear equations always has a non-zero solution if the number of unknowns is _______________.
(5) **Theorem:** The equation $A\underline{x} = \underline{b}$ has a solution if and only if $\underline{b}$ is in the linear span of ________
(6) **Theorem:** Let A be an $n \times n$ matrix and $\underline{b} \in \mathbb{R}^n$. The equation $A\underline{x} = \underline{b}$ has a unique solution if and only if A is ____________.
(7) **Theorem:** If $A\underline{u} = \underline{b}$, then the set of all solutions to the equation $A\underline{x} = \underline{b}$ consists of the vectors $\underline{u} + \underline{v}$ as $\underline{v}$ ranges over all ____________
(8) **Theorem:** Let A be an $m \times n$ matrix and let E be the row-reduced echelon matrix that is row equivalent to it. Then the non-zero rows of E are a basis for ________.

Linear combinations and Linear spans

(1) **Definition:** A vector $\underline{w}$ is a <u>linear combination</u> of $\{\underline{v}_1, \dots, \underline{v}_n\}$ if _________
(2) **Theorem:** A vector $\underline{w}$ is a linear combination of $\{\underline{v}_1, \dots, \underline{v}_n\}$ if $\mathrm{Sp}(\underline{w}, \underline{v}_1, \dots, \underline{v}_n) =$ _________
(3) **Definition:** The <u>linear span</u> of $\{\underline{v}_1, \dots, \underline{v}_n\}$ consists of _________
(4) **Definition:** A set of vectors $\{\underline{v}_1, \dots, \underline{v}_n\}$ is <u>linearly independent</u> if the only solution to the equation _______________ is _______________.
(5) **Theorem:** A set of vectors $\{\underline{v}_1, \dots, \underline{v}_n\}$ is linearly independent if the dimension of $\mathrm{Sp}(\underline{v}_1, \dots, \underline{v}_n)$ _______________
(6) **Theorem:** A set of vectors is linearly dependent if and only if one of the vectors is _________ of the others.

Subspaces

(1) **Definition:** A subset W of $\mathbb{R}^n$ is a <u>subspace</u> if it satisfies the following three conditions: _______________.
(2) **Theorem:** If V and W are subspaces of $\mathbb{R}^n$ so are _______________ and _______________.
(3) **Definition:** A set of vectors $\{\underline{v}_1, \dots, \underline{v}_d\}$ is a <u>basis</u> for a subspace V of $\mathbb{R}^n$ if _______________
(4) **Definition:** The <u>dimension</u> of a subspace V of $\mathbb{R}^n$ is _______________.

Matrices

(1) If A is an $m \times n$ matrix and B is a $p \times q$ matrix, then AB exists if and only if _______________ and in that case AB is a _______________ matrix.

(2) If A is an $m \times n$ matrix and $\underline{x} \in \mathbb{R}^n$, then $A\underline{x}$ is a linear combination of the columns of A, namely $A\underline{x} =$ ________________.
(3) The columns of a product AB are ________________.
(4) **Definition 1:** An $n \times n$ matrix A is non-singular if the only solution ________________ and is singular if it is not non-singular.
(5) **Definition 2:** An $n \times n$ matrix A is singular if there exists ________________ in $\mathbb{R}^n$ such that ________________ and is non-singular otherwise.
(6) **Theorem:** An $n \times n$ matrix A is non-singular if and only if it has ________________.
(7) **Theorem:** An $n \times n$ matrix A is singular if its columns ________________.
(8) **Theorem:** An $n \times n$ matrix A is singular if and only if its range ________________.
(9) **Theorem:** An $n \times n$ matrix A is non-singular if and only if the equation $A\underline{x} = \underline{b}$ ________________.

Invertible matrices and determinants

(1) **Definition:** An $n \times n$ matrix A is invertible if ________________.
(2) **Theorem:** An $n \times n$ matrix is invertible if and only if it is ____________.
(3) **Theorem:** An $n \times n$ matrix is invertible if and only if its ____________ is non-zero.
(4) **Theorem:** The matrix $\begin{pmatrix} w & x \\ y & z \end{pmatrix}$ is invertible if and only if ________ $\neq 0$.
(5) **Theorem:** If the matrix $\begin{pmatrix} w & x \\ y & z \end{pmatrix}$ is invertible its inverse is ____________.
(6) **Definition:** Let A be an $n \times n$ matrix. The characteristic polynomial of A is ________________
(7) **Theorem:** Let A be an $n \times n$ matrix. If B is obtained from A by
   (a) replacing row i by row i + row k with $k \neq i$, then $\det B =?$
   (b) swapping two rows of A, then $\det B =?$
   (c) multiplying a row in A by $c \in \mathbb{R}$, then $\det B =?$

Rank and Nullity

(1) **Definition:** The rank of a matrix A is the number of non-zero ________________.
(2) **Theorem:** The rank of a matrix is equal to the dimension of ________________.
(3) **Definition:** The rank of a linear transformation T is equal to ________.

Eigenvalues and eigenvectors

(1) **Definition:** Let A be an $n \times n$ matrix. We call $\lambda \in \mathbb{R}$ an eigenvalue of A if ________________
(2) **Definition:** Let A be an $n \times n$ matrix. A non-zero vector $\underline{x} \in \mathbb{R}^n$ is an eigenvector for A if ________________
(3) **Definition:** Let λ be an eigenvalue for the $n \times n$ matrix A. The λ-eigenspace for A is the set

$$E_\lambda := \{\_\_\_ \mid \_\_\_\_\_\_\_\_\_\}.$$

(4) **Theorem:** Let λ be an eigenvalue for A. The λ-eigenspace of A is a subspace of $\mathbb{R}^n$ because it is equal to the null space of __________.
(5) **Theorem:** The λ-eigenspace of A is non-zero if and only if the matrix __________ is singular.
(6) **Theorem:** If $\{\underline{v}_1, \dots, \underline{v}_n\}$ are eigenvectors for an $n \times n$ matrix A having n different eigenvalues, then ________________
(7) **Theorem:** The eigenvalues of a matrix A are the zeroes of ________________.

(8) **Theorem:** Let $\lambda_1, \dots, \lambda_r$ be different eigenvalues for a matrix A. If $\underline{v}_1, \dots, \underline{v}_r$ are non-zero vectors such that $\underline{v}_i$ is an eigenvector for A with eigenvalue λ_i, then $\{\underline{v}_1, \dots, \underline{v}_r\}$ is _________.

Linear transformations

(1) **Definition:** Let V be a subspace of $\mathbb{R}^n$ and W a subspace of $\mathbb{R}^m$. A function $T : V \to W$ is a linear transformation if ________
(2) **Definition:** The range of a linear transformation $T : V \to W$ is

$$\mathcal{R}(T) := \{\_\_ \mid \_\_\_\_\_\_\_\_\_\}.$$

(3) **Definition:** The null space of a linear transformation $T : V \to W$ is

$$\mathcal{N}(T) := \{\_\_ \mid \_\_\_\_\_\_\_\_\_\}.$$

(4) **Theorem:** Let $T : \mathbb{R}^p \to \mathbb{R}^q$ be a linear transformation. Then there is a unique ________ matrix A such that ________ for all ________. We call A the matrix that represents T.
(5) **Theorem:** The j^{th} column of the matrix representing T is _________.
(6) **Theorem:** Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation.
Then $\dim \mathcal{R}(T) + \dim \mathcal{N}(T) =$ _________.
(7) **Theorem:** Let $T : \mathbb{R}^n \to \mathbb{R}^m$ and $S : \mathbb{R}^m \to \mathbb{R}^\ell$ be linear transformations. If A represents S and B represents T, then _________ represents the composition _________.
(8) **Theorem:** Let $T : \mathbb{R}^n \to \mathbb{R}^m$ and $S : \mathbb{R}^n \to \mathbb{R}^m$ be linear transformations. If A represents S and B represents T, then _________ represents $S + T$.

Similar Matrices

(1) **Definition:** Two $n \times n$ matrices A and B are similar if _______________
(2) **[4 points]**
Theorem: If A and B are similar they have the same
(a) _______________
(b) _______________
(c) _______________
(d) _______________
(3) **Definition:** An $n \times n$ matrix A is diagonalizable if _______________
(4) **Theorem:** An $n \times n$ matrix is diagonalizable if and only if _______________.
(5) **Theorem:** Let A be an $n \times n$ matrix. If A has _______________ different _______________ it is diagonalizable.