Math 308 **Final** **August 17, 2012**

Instructions. There are three parts to the exam. Part A consists of questions that require a short answer. There is no need to show your work. In Parts A and B you get 1 point per question or, for a multi-part question, 1 point for each part. Part C consists of true/false questions. Use the scantron/bubble sheet with the convention that A =True and B =False. You will get

- +1 for each correct answer,
- -1 for each incorrect answer, and
- 0 for no answer at all.

If I have any difficulty reading your writing I will deduct points.

Part A.

Short answer questions

(1) Let S denote the set of solutions to the equation $A\underline{x} = \underline{0}$ and T the set of solutions to the equation $A\underline{x} = \underline{b}$. Describe the relation between S and T: If $A\underline{u} = \underline{b}$, then $T = \{\ldots \mid \ldots\}$. Your answer should involve $\underline{u}$ and S and the symbol $\in$ and some more.

Answer: $T = \{\underline{u} + \underline{v} \mid \underline{v} \in S\}$.

Comments:

(2) Let A be an $m \times n$ matrix with columns $\underline{A}_1, \ldots, \underline{A}_n$. Express $A\underline{x}$ as a linear combination of the columns of A.

Answer: $A\underline{x} = x_1\underline{A}_1 + \ldots + x_n\underline{A}_n$

Comments:

(3) Write down a 4×3 matrix A such that $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2z \\ y-z \\ x-y+z \\ x-z \end{pmatrix}$.

Answer: $\begin{matrix} 0 & 0 & 2 \\ 0 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 0 & -1 \end{matrix}$

Comments:

(4) Let $\underline{A}_1, \underline{A}_2, \underline{A}_3, \underline{A}_4$ be the columns of a 4×4 matrix A and suppose that $2\underline{A}_1 + 2\underline{A}_4 = \underline{A}_3 - 3\underline{A}_2$. Write down a solution to the equation $A\underline{x} = \underline{0}$ of

the form

$$\underline{x} = \begin{pmatrix} 6 \\ ? \\ ? \\ ? \end{pmatrix}$$

Answer:

Comments:

(5) $(3,1,1,3)^T$ and $(1,2,2,1)^T$ are solutions to the two (different!) homogeneous equations _________ and _________

Answer:

Comments:

(6) $(3,1,1,3)^T$ and $(1,2,2,1)^T$ belong to the 2-dimensional subspace of $\mathbb{R}^4$ that is the set of all solutions to the equations _________

(7) Find a basis for the line $x_1 + x_2 = x_2 + x_3 = 3x_1 - 2x_4 = 0$ in $\mathbb{R}^4$.

Answer:

Comments:

(8) Find two linearly independent vectors that lie on the plane in $\mathbb{R}^4$ given by the equations

$$2x_1 - x_2 + x_3 - x_4 = 0$$
$$3x_1 - x_2 + x_3 - x_4 = 0$$

Answer:

Comments:

(9) If $A^T A\underline{x} = A^T \underline{b}$, then $\underline{x}$ is a _________ .

Answer:

Comments:

(10) The matrix representing the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that rotates a vector by θ radians in the counter-clockwise direction is _________.

Answer:

Comments:

Part B.

Complete the definitions and Theorems.
There is a difference between theorems and definitions.
Don't write the part of the question I have already written. Just fill in the blank.

(1) **Definition:** A vector $\underline{x}$ is a linear combination of $\underline{v}_1, \dots, \underline{v}_n$ if ________

Answer: $\underline{x} = a_1\underline{v}_1 + \cdots + a_n\underline{v}_n$ for some $a_1, \dots, a_n \in \mathbb{R}$.

Comments: Don't say "where" where I have written "for some".

(2) **Definition:** The linear span of $\underline{v}_1, \dots, \underline{v}_n$ is the set of all ________

Answer: linear combinations of $\underline{v}_1, \dots, \underline{v}_n$.

Comments: Don't say "all possible". That means the same as "all". Keep your answer as simple as possible. In that spirit, there is no need to insert the word "vectors" before $\underline{v}_1, \dots, \underline{v}_n$. It is implicit in the question that the $\underline{v}_i$'s are vectors.

(3) **Definition:** A set of vectors $\{\underline{v}_1, \dots, \underline{v}_n\}$ is linearly independent if the only solution to the equation ________

Answer: $a_1\underline{v}_1 + \cdots + a_n\underline{v}_n = \underline{0}$ is $a_1 = \cdots = a_n = 0$.

Comments: Don't say "$a_1\underline{v}_1 + \cdots + a_n\underline{v}_n = \underline{0}$ is the trivial solution". Better you should make specific what you mean by the trivial solution.

One "answer" was "$A\underline{v} = \underline{0}$ is the trivial solution $\underline{v} = \underline{0}$". This person is confused. What is A here?

(4) **Definition:** $\{\underline{v}_1, \dots, \underline{v}_n\}$ is linearly dependent if ________ $= \underline{0}$ for some ________

Answer: $a_1\underline{v}_1 + \cdots + a_n\underline{v}_n = \underline{0}$ for some numbers $a_1, \dots, a_n$ that are not all zero.

Comments: It is essential that you include the phrase "not all zero".

(5) **Definition:** Let V be a subspace of $\mathbb{R}^n$ and W a subspace of $\mathbb{R}^m$. A function $T : V \to W$ is a linear transformation if $T(a_1\underline{x}_1 + \cdots + a_k\underline{x}_k) =$ ________ for all ________.

Answer: $a_1T(\underline{x}_1) + \cdots + a_kT(\underline{x}_k)$ for all $a_1, \dots, a_k \in \mathbb{R}$ and all $\underline{x}_1, \dots, \underline{x}_k \in V$.

Comments: Some wrote $a_1T(\underline{x}_1) + \cdots + a_kT(\underline{x}_k)$ for all $a \in \mathbb{R}$ and $\underline{x} \in V$. That leaves the reader to figure out what you really meant because there is no a and no $\underline{x}$ mentioned in the question. There are numbers labelled $a_1, \dots, a_k$ and vectors labelled $\underline{x}_1, \dots, \underline{x}_k$.

(6) **Definition:** The norm or length of a vector $\underline{v}$ is _________.

Answer:

Comments:

(7) **Definition:** A set of vectors $\{\underline{v}_1, \ldots, \underline{v}_n\}$ is orthogonal if _________.

Answer:

Comments:

(8) **Definition:** A set of vectors $\{\underline{v}_1, \ldots, \underline{v}_n\}$ is orthonormal if _________.

Answer:

Comments:

(9) **Definition:** The null space or kernel and range of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ are
(a) $\ker(T) = \{\cdots \mid \cdots\}$ and
(b) $\mathcal{R}(T) = \{\cdots \mid \cdots\}$.

Answer:

Comments:

(10) **Definition:** A subset W of $\mathbb{R}^n$ is a subspace if it is non-empty and
(a) _________
(b) _________

Answer:

Comments:

(11) **Definition:** A set of vectors $\{\underline{v}_1, \ldots, \underline{v}_d\}$ is a basis for a subspace V of $\mathbb{R}^n$ if _______________

Answer:

Comments:

(12) **Definition:** The dimension of a subspace V of $\mathbb{R}^n$ is _______________.

Answer:

Comments:

(13) **Definition:** A function $f : X \to Y$ is one-to-one if _______________.

Answer: $f(x) = f(x')$ if and only if $x = x'$.

Comments: Some people wrote $\underline{x}$. This *not* a question about vectors. It is a question about arbitrary functions. For example X might be the set of leaves and Y the set of trees and f the function that sends a leaf to the tree it grew on. Or, X might be a set of people and Y might be $\mathbb{R}^3$ and $f(x)$ might be (the age of x, the height of x, the weight of x).

(14) **Definition:** The range of a function $f : X \to Y$ is ______________.

Answer:

Comments:

(15) **Definition:** A function $f : X \to Y$ is onto if ______________.

Answer:

Comments:

(16) **Definition:** Functions $f : X \to Y$ and $g : Y \to X$ are each other's inverse if ______________.

Answer:

Comments:

(17) **Theorem:** A function $f : X \to Y$ has an inverse if and only if ______________.

Answer: f is one-to-one and onto.

Comments:

(18) **Theorem:** If $T : \mathbb{R}^p \to \mathbb{R}^q$ is a linear transformation the dimensions of $\mathcal{R}(T)$ and $\ker(T)$ are related by the formula ______________.

Answer: $\dim \ker(T) + \dim \mathcal{R}(T) = p$.

Comments: Many people wrote $\ker(T) + \mathcal{R}(T) = p$ but this is nonsense: $\ker(T)$ is not a number; $\mathcal{R}(T)$ is not a number; we can add subspaces *but only if they are contained in a common* $\mathbb{R}^n$; here $\ker(T) \subset \mathbb{R}^p$ and $\mathcal{R}(T) \subset \mathbb{R}^p$. We can't add a subspace of (or vector in) $\mathbb{R}^p$ to a subspace of (or vector in) $\mathbb{R}^q$.

(19) **Theorem:** The following conditions on an $n \times n$ matrix A are equivalent:
- (a) A is invertible
- (b) the equation $A\underline{x} = \underline{b}$ has ___________
- (c) the columns of A are ___________
- (d) the rows of A are ___________
- (e) $\mathcal{R}(A) =$ ___________

(f) $\mathcal{N}(A) =$____________
(g) $\operatorname{rank}(A) =$ ____________
(h) A^T is ____________
(i) $\det(A)$ is ____________

Answer:

Comments:

(20) **Theorem:** A set of vectors is linearly dependent if and only if one of the vectors is _________ of the others.

Answer:

Comments:

(21) **Definition:** Let A be an $n \times n$ matrix. We call $\lambda \in \mathbb{R}$ an eigenvalue of A if _______________

Answer:

Comments:

(22) **Definition:** Let A be an $n \times n$ matrix. A non-zero vector $\underline{x} \in \mathbb{R}^n$ is an eigenvector for A if _______________

Answer:

Comments:

(23) **Definition:** Let λ be an eigenvalue for the $n \times n$ matrix A. The λ-eigenspace for A is the set

$$E_\lambda := \{\_\_ \mid \_\_\_\_\_\_\_\_\_\}.$$

Answer:

Comments:

(24) **Theorem:** Let λ be an eigenvalue for A. The λ-eigenspace of A is a subspace of $\mathbb{R}^n$ because it is equal to the null space of _________.

Answer:

Comments:

(25) **Theorem:** The λ-eigenspace of A is non-zero if and only of the determinant of _________.

Answer:

Comments:

(26) **Theorem:** The eigenvalues of a matrix A are the roots of ______________.

Answer:

Comments:

(27) **Theorem:** Let $\lambda_1, \dots, \lambda_r$ be different eigenvalues for a matrix A. If $\underline{v}_1, \dots, \underline{v}_r$ are non-zero vectors such that $\underline{v}_i$ is an eigenvector for A with eigenvalue λ_i, then $\{\underline{v}_1, \dots, \underline{v}_r\}$ is _________.

Answer:

Comments:

(28) **Theorem:** Let $T : \mathbb{R}^p \to \mathbb{R}^q$ be a linear transformation. Then there is a unique ________ matrix A such that ________ for all ________. We call A the matrix that <u>represents</u> T.

Answer:

Comments:

(29) **Theorem:** The j^{th} column of the matrix representing T is _________.

Answer:

Comments:

(30) **Theorem:** Let W be a subspace of $\mathbb{R}^n$ and $W^\perp$ its orthogonal. Every $\underline{v} \in \mathbb{R}^n$ can be written in a unique way as a sum of ________ and ________.

Answer:

Comments:

(31) Let W be a subspace of $\mathbb{R}^n$ and $P : \mathbb{R}^n \to \mathbb{R}^n$ the orthogonal projection onto W. Let $\underline{v} \in \mathbb{R}^n$. How is $P(\underline{v})$ related to your answer to the previous question?

Answer:

Comments:

(32) **Definition:** Let A be an $m \times n$ matrix. A vector $\underline{x}^*$ in $\mathbb{R}^n$ is a <u>least-squares</u> solution to $A\underline{x} = \underline{b}$ if ________.

Answer:

Comments:

(33) **Theorem:** A vector $\underline{x}^*$ in $\mathbb{R}^n$ is a least-squares solution to $A\underline{x} = \underline{b}$ if and only if it is a solution to the equation ________.

Answer:

Comments:

(34) **Definition:** An $n \times n$ matrix Q is orthogonal if ________.

Answer:

Comments:

(35) **Theorem:** The following 3 conditions on an $n \times n$ matrix Q are equivalent:
(a) Q is orthogonal;
(b) ________ for all $\underline{x} \in \mathbb{R}^n$;
(c) ________ for all $\underline{u}, \underline{v} \in \mathbb{R}^n$.

Answer:

Comments:

Part C.

True or False

Remember +1, 0, or -1.

(1) The range of a non-zero linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^4$ is a subspace of $\mathbb{R}^4$ whose dimension is either 1 or 2.

True. []

Comments:

(2) The kernel of a non-zero linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^4$ is a subspace of $\mathbb{R}^2$ whose dimension is either 0 or 1.

True. []

Comments:

(3) Let V be a 4-dimensional subspace of $\mathbb{R}^5$. Every set of four vectors in V spans V.

False. []

Comments:

(4) Let V be a 4-dimensional subspace of $\mathbb{R}^5$. Every set of five vectors in V is linearly dependent.

True. []

Comments:

(5) Let V be a 4-dimensional subspace of $\mathbb{R}^5$. If W is a 2-dimensional subspace of $\mathbb{R}^5$, then $V \cap W \neq \{\underline{0}\}$.

True. []

Comments:

(6) Let V be a 4-dimensional subspace of $\mathbb{R}^5$. There is a basis for $\mathbb{R}^5$ consisting of 4 vectors in V and one vector not in V.

True. []

Comments:

(7) If V is a subspace of $\mathbb{R}^n$, then $V + V^\perp = \mathbb{R}^n$.

True. []

Comments:

(8) A 3-dimensional subspace of $\mathbb{R}^5$ contains infinitely many different subspaces of $\mathbb{R}^5$.

True. []

Comments:

(9) A 3-dimensional subspace of $\mathbb{R}^5$ has infinitely many different bases.

True. []

Comments:

(10) If A and B are $m \times n$ matrices such that B can be obtained from A by elementary row operations, then A can also be obtained from B by elementary row operations.

True. []

Comments:

(11) If $\underline{u}$, $\underline{v}$, and $\underline{w}$, are any vectors in $\mathbb{R}^4$, then $\{3\underline{u} - 2\underline{v}, 2\underline{v} - 4\underline{w}, 4\underline{w} - 3\underline{u}\}$ is linearly dependent.

True. []

Comments:

(12) Every set of orthogonal vectors in $\mathbb{R}^n$ is linearly independent.

True. []

Comments:

(13) Let A be an $m \times n$ matrix. If $A^T A = I$, then the columns of A form an orthonormal set.

True. []

Comments:

(14) If $A = BC$, then every solution to $C\underline{x} = \underline{0}$ is a solution to $A\underline{x} = \underline{0}$.

True. []

Comments:

(15) Let A be an $m \times n$ matrix and B an $n \times p$ matrix. If $C = AB$, then $\{C\underline{x} \mid \underline{x} \in \mathbb{R}^p\} \subset \{A\underline{w} \mid \underline{w} \in \mathbb{R}^n\}$.

True. []

Comments:

(16) The linear span of the vectors $(4, 0, 0, 1)$, $(0, 3, 2, 0)$ and $(4, 3, 2, 1)$ is the 3-plane $x_1 - 2x_2 + 3x_3 - 4x_4 = 0$ in $\mathbb{R}^4$.

False. []

Comments:

(17) The linear span of the vectors $(4, 0, 0, 1)$, $(0, 2, 0, -1)$ and $(4, 3, 2, 1)$ is the 3-plane $x_1 - 2x_2 + 3x_3 - 4x_4 = 0$ in $\mathbb{R}^4$.

True. []

Comments:

(18) Let $\underline{u}$, $\underline{v}$, and $\underline{w}$ be vectors in $\mathbb{R}^n$. Then $\mathsf{span}\{\underline{u}, \underline{v}, \underline{w}\} = \mathsf{span}\{\underline{u} + 2\underline{v}, \underline{u} + 3\underline{v}, \underline{u} + \underline{v} + \underline{w}\}$.

True. []

Comments:

(19) The set $\{x_1, x_2, x_3, x_4) \mid x_1 + x_3 = x_2 - x_4 = 0\}$ is a subspace of $\mathbb{R}^4$.

True. []

Comments:

(20) The set $\{x_1, x_2, x_3, x_4) \mid x_1 + x_3 - x_2 + x_4 = 1\}$ is a subspace of $\mathbb{R}^4$.

False. []

Comments:

(21) Every subspace of $\mathbb{R}^n$ has an orthogonal basis.

True. []

Comments:

(22) Every subspace of $\mathbb{R}^n$ has an orthonormal basis.

True. []

Comments:

(23) $\text{span}\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 5 \\ 6 \\ 7 \\ 8 \end{pmatrix}, \begin{pmatrix} 9 \\ 10 \\ 11 \\ 12 \end{pmatrix} \right\} = \text{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix} \right\}.$

True. []

Comments:

(24) Every subset of a linearly independent set is linearly independent.

True. []

Comments:

(25) The dimension of a subspace is the number of elements in it.

False. []

Comments:

(26) Let X and Y be subsets of $\mathbb{R}^n$. If $X \subset Y$ and X is linearly dependent, then Y is linearly dependent.

True. []

Comments:

(27) The row reduced echelon form of a square matrix is the identity if and only if the matrix is invertible.

True. []

Comments:

(28) The row and column spaces of a matrix always have the same dimension.

True. []

Comments:

(29) If A and B are $m \times n$ matrices such that B can be obtained from A by elementary row operations, then A can also be obtained from B by elementary row operations.

True. []

Comments:

(30) For all matrices A and B, $\mathcal{R}(AB) \subset \mathcal{R}(A)$

True. []

Comments:

(31) Every 5×5 matrix has an eigenvector. (Hint: think of the graph of its characteristic polynomial.)

True. []

Comments:

(32) Every 6×6 matrix has an eigenvector.

False. []

Comments:

(33) The vectors $(2, 2, -4, 3, 0)$ and $(0, 0, 0, 0, 1)$ are a basis for the subspace $x_1 - x_2 = 2x_2 + x_3 = 3x_1 - 2x_4 = 0$ of $\mathbb{R}^5$.

True. []

Comments:

(34) The vectors $(1, 1)$, $(1, -2)$, $(2, -3)$ are a basis for the subspace of $\mathbb{R}^4$ that is the set of solutions to the equations $x_1 - x_2 = 2x_2 + x_3 = 3x_1 - 2x_4 = 0$.

False. []

Comments:

(35) $\{\underline{x} \in \mathbb{R}^4 \mid x_1 - x_2 = x_3 + x_4\}$ is a subspace of $\mathbb{R}^4$.

True. []

Comments:

(36) $\{\underline{x} \in \mathbb{R}^5 \mid x_1 - x_2 = x_3 + x_4 = 1\}$ is a subspace of $\mathbb{R}^5$.

False. []

Comments:

(37) The set $\{\underline{x} \in \mathbb{R}^4 \mid x_1 = x_2\} \cup \{\underline{x} \in \mathbb{R}^4 \mid x_3 = x_4\}$ is a subspace of $\mathbb{R}^4$.

False. []

Comments:

(38) The set $\{\underline{x} \in \mathbb{R}^4 \mid x_1 = x_2\} \cap \{\underline{x} \in \mathbb{R}^4 \mid x_3 = x_4\}$ is a subspace of $\mathbb{R}^4$.

True. []

Comments:

(39) The equation $A\underline{x} = \underline{b}$ has a solution if and only if $\underline{b}$ is linear combination of the columns of A.

True. []

Comments:

(40) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y - z \\ z - x \\ x - y \end{pmatrix}$. The kernel (null space) of T is $\left\{ \begin{pmatrix} t \\ t \\ t \end{pmatrix} \,\middle|\, t \in \mathbb{R} \right\}$.

True. []

Comments:

(41) Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be the linear transformation $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \\ x_2 \\ x_2 \end{pmatrix}$. The vectors $\begin{pmatrix} -2 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ are a basis for the range of T.

True. []

Comments:

(42) There is a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^6$ such that $\mathcal{R}(T) = \mathcal{N}(T)$.

False. []

Comments:

(43) There is a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^6$ such that $\dim \mathcal{R}(T) = \dim \mathcal{N}(T)$.

True. []

Comments:

(44) Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation $T(x_1, x_2, x_3, x_4) = (0, x_1, x_2, x_3)$. The null space of T is $\{(a, 0, 0, 0) \mid a \in \mathbb{R}\}$.

False. []

Comments:

(45) Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation $T(x_1, x_2, x_3, x_4) = (0, x_1, x_2, x_3)$. The null space of T is $\{(x_4, 0, 0, 0) \mid x_4 \text{ is a real number}\}$.

False. []

Comments:

(46) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation $T(x_1, x_2) = (x_2, 0)$. The null space of T is $\{(x_3, 0) \mid x_3 \in \mathbb{R}\}$.

False. []

Comments:

(47) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation $T(x_1, x_2) = (x_2, 0)$. Then $\{(5, 0)\}$ is a basis for the null space of T.

True. []

Comments:

(48) The set the set $\{(-2, 0, 0, 0), (0, 0, 1, 1)\}$ is a basis for the kernel of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ defined by $T(x_1, x_2, x_3) = (x_1, 0, x_2, x_2)$.

False. []

Comments:

(49) The set $\{(2, 0, 0, 0), (0, 0, 3, 3)\}$ is a basis for the range of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ given by $T(x_1, x_2, x_3) = (x_1, 0, x_2, x_2)$.

True. []

Comments:

(50) If V and W are subspaces of $\mathbb{R}^n$ so is $\{\underline{v} + \underline{w} \mid \underline{v} \in V \text{ and } \underline{w} \in W\}$.

True. []

Comments:

(51) If V and W are subspaces of $\mathbb{R}^n$ so is $\{2\underline{v} - 3\underline{w} \mid \underline{v} \in V \text{ and } \underline{w} \in W\}$.

True. []

Comments:

(52) Every non-zero linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^1$ is onto.

True. []

Comments:

(53) There is a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ that is onto.

True. []

Comments:

(54) There is a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ that is one-to-one.

False. []

Comments:

(55) Let $\underline{u}$ and $\underline{v}$ be linearly independent vectors that belong to the subspace of $\mathbb{R}^4$ consisting of solutions to the homogeneous system of equations

$$4x_1 - 3x_2 + 2x_3 - x_4 = 0$$
$$x_1 + 2x_2 + 3x_3 + 4x_4 = 0$$

The vector $(1, 2, 3, 4)$ is a linear combination of $\underline{u}$ and $\underline{v}$.

False. []

Comments:

(56) Let $\underline{u}$ and $\underline{v}$ be linearly independent vectors that belong to the subspace of $\mathbb{R}^4$ consisting of solutions to the homogeneous system of equations

$$4x_1 - 3x_2 + 2x_3 - x_4 = 0$$
$$2x_1 + 3x_2 - 4x_3 + x_4 = 0$$

The vector $(1, 2, 3, 4)$ is a linear combination of $\underline{u}$ and $\underline{v}$.

True. []

Comments:

(57) If A and B are $n \times n$ matrices, then $\det(AB) = \det(A)\det(B)$.

True. []

Comments:

(58) If A and B are $n \times n$ matrices, then $\det(A + B) = \det(A) + \det(B)$.

False. []

Comments:

In the next 4 questions, A is a 4×4 matrix whose columns $\underline{A}_1, \underline{A}_2, \underline{A}_3, \underline{A}_4$ have the property that $\underline{A}_1 + \underline{A}_2 = \underline{A}_3 + \underline{A}_4$.

(59) The rows of A are linearly dependent.

True. []

Comments:

(60) The rank of A must be 3.

False. []

Comments:

(61) $A \begin{pmatrix} 2 \\ 3 \\ 1 \\ 4 \end{pmatrix} = \underline{0}$.

True. []

Comments:

(62) $A \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} = \underline{0}$.

True. []

Comments: