Math 308 **Week 3 Quiz 1**

Complete the sentence/definition:

(1) If a system of linear equations has ≥ 2 solutions, then _____ solutions.
(2) A system of linear equations can have either
   (a) a _____ solution, or
   (b) _____ solutions, or
   (c) _____ solutions.
(3) An $m \times n$ system of linear equations consists of
   (a) ___ linear equations
   (b) in ___ unknowns.
(4) If an $m \times n$ system of linear equations is converted into a single matrix equation $A\underline{x} = \underline{b}$, then
   (a) A is a ___× ___ matrix
   (b) $\underline{x}$ is a ___× ___ matrix
   (c) $\underline{b}$ is a ___× ___ matrix
(5) A system of linear equations is consistent if _____
(6) A system of linear equations is inconsistent if _____
(7) An $m \times n$ matrix has
   (a) ___ columns and
   (b) ___ rows.
(8) The solutions to a system of m linear equations in n unknowns are points in _____
(9) If p and q are different solutions to a system of linear equations so are _____
(10) A vector $\underline{w}$ is a linear combination of $\{\underline{v}_1, \ldots, \underline{v}_n\}$ if there are _____.
(11) The equation $A\underline{x} = \underline{b}$ has a solution if and only if $\underline{b}$ is a _____.
(12) The linear span of $\{\underline{v}_1, \ldots, \underline{v}_n\}$ consists of _____.
(13) The rank of a matrix is the number _____.
(14) The rank of an $m \times n$ matrix is $\leq$ _____.
(15) Let A be an $m \times n$ matrix having rank r. If $r = n$, then the equation $A\underline{x} = \underline{b}$ has _____.
(16) A set of vectors $\{\underline{v}_1, \ldots, \underline{v}_n\}$ is linearly dependent if the equation _____ has a solution such that _____
(17) A set of vectors $\{\underline{v}_1, \ldots, \underline{v}_n\}$ is linearly independent if the only solution to_____
(18) Write down 3 linearly independent vectors in $\mathbb{R}^4$.
(19) If

$$A\begin{pmatrix}1\\2\\3\\4\end{pmatrix} = \begin{pmatrix}7\\8\\9\end{pmatrix} \qquad \text{and} \qquad A\begin{pmatrix}2\\2\\2\\2\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix},$$

then

$$A\begin{pmatrix}4\\6\\8\\10\end{pmatrix} = ?,$$

(20) The free, or independent, variables in the system of linear equations

$$\begin{pmatrix} 1 & 0 & 2 & 0 & 0 & 3 & 4 \\ 0 & 1 & 3 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

are _____.

(21) In the previous question, express one of the dependent variables in terms of the independent variables.

(22) **True or False:** $\{(1,2),(2,3)\}$ is linearly dependent.

(23) **True or False:** Every vector in $\mathbb{R}^2$ is a linear combination of $\{(1,2),(2,3)\}$.

(24) **True or False:** A matrix is row equivalent to a unique row reduced echelon matrix.

(25) **True or False:** An $m \times n$ system of homogeneous equations has a unique solution if $m < n$.

(26) Is $\mathbb{R}^4$ equal to span$\{(1,1,0,0),(0,1,1,0),(0,0,1,1),(1,1,1,1)\}$? Why?

(27) Are the vectors $(1,1,0,0)$, $(0,1,1,0)$, $(0,0,1,1)$, and $(1,1,1,1)$, linearly dependent? Why?

(28) Is $\mathbb{R}^4$ equal to span$\{(1,0,0,0),(0,2,0,0),(0,0,3,0),(0,0,0,4)\}$? Why?

(29) A matrix E is in row echelon form if

(a) _____

(b) _____

(c) _____

(30) A matrix is in row reduced echelon form if it is _____ and _____ .

(31) In this question use R_i to denote the i^{th} row of a matrix. The three elementary row operations are

(a) _____

(b) _____

(c) _____