Math 308 **Week 3 Quiz 2**

(1) Describe in geometric language the set $A = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$. Your answer should begin "A is the ..."
(2) The system of equations $A\underline{x} = \underline{b}$ is homogeneous if _________
(3) The set of solutions to a 3×3 system of linear equations is one of the four possibilities in the previous answer or _________.
(4) Let $\underline{u}$ be a solution to the equation $A\underline{x} = \underline{b}$. Then every solution to $A\underline{x} = \underline{b}$ is of the form _________ where _________
(5) In the previous question let S denote the set of solutions to the equation $A\underline{x} = \underline{0}$ and T the set of solutions to the equation $A\underline{x} = \underline{b}$. If $A\underline{u} = \underline{b}$, then
$$T = \{\dots \mid \dots\}.$$
Your answer should involve $\underline{u}$ and S and the symbol $\in$ and some more.
(6)
$$\text{If } B\begin{pmatrix}1\\2\\3\\4\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \text{ and } \begin{pmatrix}4\\3\\2\\1\end{pmatrix} \text{ is a solution to the equation } B\underline{x} = \begin{pmatrix}1\\0\\-1\end{pmatrix},$$
$$\text{then } \underline{\ ??\ } \text{ is also a solution to the equation } B\underline{x} = \begin{pmatrix}1\\0\\-1\end{pmatrix}.$$
(7) The system of equations $A\underline{x} = \underline{b}$ is homogeneous if _________
(8) I want a geometric description of the solutions: the set of solutions to a 2×3 system of linear equations is either
  (a) _________
  (b) or _________
  (c) or _________
  (d) or _________
(9) The set of solutions to a 3×3 system of linear equations is one of the four possibilities in the previous answer or _________.
(10) Let $\underline{u}$ be a solution to the equation $A\underline{x} = \underline{b}$. Then every solution to $A\underline{x} = \underline{b}$ is of the form _________ where _________
(11) In the previous question let S denote the set of solutions to the equation $A\underline{x} = \underline{0}$ and T the set of solutions to the equation $A\underline{x} = \underline{b}$. If $A\underline{u} = \underline{b}$, then
$$T = \{\dots \mid \dots\}.$$
Your answer should involve $\underline{u}$ and S and the symbol $\in$ and some more.
(12) A matrix E is in row echelon form if
  (a) _________
  (b) _________
  (c) _________
(13) A matrix is in row reduced echelon form if it is _________ and _________ .
(14) In this question use R_i to denote the i^{th} row of a matrix. The three elementary row operations are
  (a) _________
  (b) _________
  (c) _________

(15) If $(A|\underline{b})$ is the augmented matrix for an $m \times n$ system of linear equations and is in reduced echelon form, then the system of linear equations is inconsistent if _________

(16) The system of linear equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \qquad\qquad\qquad\qquad &\quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m. \end{aligned}$$

is <u>homogeneous</u> if _________

(17) True or False: The <u>row echelon form</u> of a matrix is unique.

(18) True or False: The <u>row reduced echelon form</u> of a matrix is unique.

(19) True or False: Given a matrix A there is a <u>unique</u> matrix in row echelon form that is equivalent to A.

(20) True or False: Given a matrix A there is a <u>unique</u> matrix in row reduced echelon form that is equivalent to A.

(21) A solution to a <u>homogeneous</u> system of equations is <u>non-trivial</u> if _________

(22) If $\underline{u}$, $\underline{v}$, and $\underline{w}$, are elements of a vector space V, then $\{\underline{u}-\underline{v}, \underline{v}-\underline{w}, \underline{w}-\underline{u}\}$ is linearly dependent because _________. (Your answer should consist of a single equation.)

(23) Write down a non-zero point in $\mathbb{R}^5$ of the form $(0,0,0,-,-)$ that is a solution to the equation

$$5x_1 - 4x_2 + 3x_3 - 2x_4 + x_5 = 0.$$

(24) Write down a non-zero point in $\mathbb{R}^5$ of the form $(0,0,-,0,-)$ that is a solution to the equation

$$5x_1 - 4x_2 + 3x_3 - 2x_4 + x_5 = 0.$$

(25) Write down a non-zero point in $\mathbb{R}^5$ of the form $(0,-,0,0,-)$ that is a solution to the equation

$$5x_1 - 4x_2 + 3x_3 - 2x_4 + x_5 = 0.$$

(26) Write down a non-zero point in $\mathbb{R}^5$ of the form $(-,0,0,0,-)$ that is a solution to the equation

$$5x_1 - 4x_2 + 3x_3 - 2x_4 + x_5 = 0.$$

(27) Are the vectors in your answers to the last four questions linearly independent? Why?

(28) Explain the following joke. On the linear algebra exam the professor writes down two vectors and asks if they are linearly dependent. The student answers "the first is but second is not."

(29) Find equations for a plane in $\mathbb{R}^4$ that contains $(0,0,0,0)$ and $(1,0,1,0)$ and $(0,1,2,3)$.

(30) Do the vectors $(1,0,0,0)$, $(0,1,0,0)$, $(0,0,1,0)$, and $(0,0,0,1)$, span $\mathbb{R}^4$? Why?

(31) Are the vectors $(1,2,3,4)$, $(5,6,7,8)$, $(12,11,10,9)$, $(2,4,8,16)$, and $(1,1,1,1)$, linearly dependent. Why?