RAINWATER SEMINAR

Ergodic Theory and Dynamical Systems, Harmonic and Functional Analysis

Date	Tuesday, February 15
Time	2:30 p.m.
Room	Padelford C-401
Speaker	Boris Solomyak
Title	Does positive Lebesgue measure imply nonempty interior for self-similar sets?

Abstract

A nonempty compact set $E \subset \mathbb{R}$ is *self-similar* if it can be written as $E = \bigcup_{i=1}^{m} f_i(E)$, where $m \geq 2$, $f_i(x) = \lambda_i x + d_i$, $0 < \lambda_i < 1$, and $d_i \in \mathbb{R}$. Affine middle-α Cantor sets are well-known examples. It is an open problem whether positive Lebesgue measure implies nonempty interior for self-similar sets on the line. It is related to another open problem, raised by J. Palis and F. Takens, whether such an implication holds for arithmetic sums of affine Cantor sets.

One may also ask what happens in higher dimensions. Recently, an example was found, jointly with M. Csörnyei, T. Jordan, M. Pollicott, and D. Preiss, of a self-similar set in $\mathbb{R}^2$ with positive measure but empty interior.

In this talk I will survey known results on the line, focusing on self-similar sets, and describe the planar example.