Math 534 **HOMEWORK 8 (due Wednesday, Nov 21)** Autumn 2007

1-2 (4 pts) Do problems 1(bc) and 2(bc) from Gamelin VI.1 (p. 171).

3 (4 pts) Find all singularties of all functions below. (Caution: some functions might not be well-defined, and some might just look as if they are not well-defined.) Determine the nature of these singularities (i.e. decide whether they are isolated singularities and classify them as removable, pole or essential). In caser of isolated singularities, find the "radius of convergence" of the Laurent series at the singularity. In case of poles, determine their order.

(a) $z/\tan^2(z)$ (b) $z\sin(1/z)$ (c) $ze^z/(z^2+1)$

(d) $\cos\sqrt{z}$ (e) $(\log z)/(1+z)^2$ (f) $z/\sin\sqrt{z}$

4 (3 pts) Do problem 11 from Gamelin VI.2 (p. 177).

5 (2 pts) Do problem 12 from Gamelin VI.2 (p. 177).

6 (3 pts) Prove that there is no function analytic in the unit disk such that $|f(z)| \to \infty$ as $|z| \to 1$.

7 (2 pts) Suppose $K > 1$. Find all solutions to the functional equation $f(Kz) = zf(z)$ which are analytic in $\mathbb{C} \setminus \{0\}$.

8 (2 pts) Suppose f is a one-to-one analytic function mapping $\mathbb{C}$ onto $\mathbb{C}$. Prove that $f(z) = az + b$ for some constants a and b. *Hint:* what kind of singularity can f have at ∞?