PREREQUISITES

Know what the following mean (review them tonight). If you have only seen the corresponding proofs for real numbers and real-valued functions, over the next week check to see if the same proofs also work for "real" replaced by "complex". If many of the concepts below are new to you, then I would recommend that you first take the senior level analysis class.

Let $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$ be sequences of real numbers and let $\{f_n\}$ be a sequence of real-valued functions defined on some interval $I \subset \mathbb{R}$.

1. $\{a_n\}$ converges to a (Notation: $a_n \to a$) "$\epsilon - \delta$ " version.
2. Cauchy sequence
3. $\sum a_n$ converges, converges absolutely (Notation: $\sum |a_n| < \infty$)
4. $\sum a_n$ converges implies $a_n \to 0$, but not conversely.
5. $\limsup_{n\to\infty} a_n$, $\liminf_{n\to\infty} a_n$
6. ratio test, comparison test, Weierstrass M-test, root test
7. can rearrange absolutely convergent series and get same sum, but can't do the same for conditionally convergent series.
8. If $\sum_{n=0}^{\infty} a_n = A$ and $\sum_{n=0}^{\infty} b_n = B$ then

$$A + B = \sum_{n=0}^{\infty}(a_n + b_n)$$

$$cA = \sum_{n=0}^{\infty} ca_n.$$

If $\sum a_n$ converges absolutely and $c_n = \sum_{k=0}^{n} a_k b_{n-k}$ then

$$AB = \sum_{n=0}^{\infty} c_n.$$

9.

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_{n,k} = \sum_{k=0}^{\infty}\sum_{n=0}^{\infty} a_{n,k}$$

provided at least one sum converges absolutely.

10. Continuous function, uniformly continuous function
11. $f_n(x) \to f(x)$ pointwise, $f_n(x) \to f(x)$ uniformly.
12. Uniform limit of a sequence of continuous functions is continuous.

13. If $f_n \to f$ uniformly on a bounded interval I then

$$\lim \int_I f_n(x)dx = \int_I \lim f_n(x)dx = \int_I f(x)dx$$

14. Cor.:

$$\sum_{n=0}^{\infty} \int_I f_n(x)dx = \int_I \sum_{n=0}^{\infty} f_n(x)dx,$$

if the partial sums of $\sum f_n$ converge uniformly on the bounded interval I.

15. open set, closed set, connected set, compact set, metric space.

16. f continuous on a compact set X implies f is uniformly continuous on X.

17. $X \subset \mathbb{R}^n$ is compact if and only if it is closed and bounded.

18. A metric space X is compact if and only if every infinite sequence in X has a limit point in X. (This can fail if X is not a metric space)

19. If f is continuous on a connected set U, then $f(U)$ is connected. If f is continuous on a compact set K then $f(K)$ is compact.

20. A continuous real-valued function on a compact set has a maximum and a minimum.

21. Green's theorem. You should look at the proof you learned (if in fact you saw a proof) and figure out exactly what the hypotheses are for that version. (many undergrad books prove a special case, then wave their hands).

All of the above can be found in the undergraduate text Rudin, Principles of Mathematical Analysis as well as many other sources. #15 -#20 are also in Ahlfors, Complex analysis, Chapter 3 section 1 (pages 50-61).