Lecture 1 (May 5) (See [4, pp.19–26])

1. Measure preserving transformations (def.), enough to check on the semi-algebra.
2. Examples: Circle rotations, compact group rotations, with the Haar measure; compact group endomorphisms with the Haar measure, the Gauss map on $[0, 1]$.
3. Associated isometries.
4. Poincaré Recurrence Theorem

Lecture 2 (May 7) (See [4, pp.26–31])

1. Ergodicity, equivalent conditions for ergodicity (e.g. no non-constant invariant measurable functions).
2. Examples: irrational circle rotations; a toral endomorphism defined by an integer matrix is ergodic iff the matrix has no eigenvalues that are roots of unity.

Lecture 3 (May 9) (See [3, pp.13–18])

1. Proof of Birkhoff's Ergodic Theorem.
2. Corollary: two ergodic invariant measures for the same transformation are mutually singular.

Lecture 4 (May 12)

1. Example: Bernoulli shift with finitely many states is ergodic (even mixing); get a family of measures ν_p on the unit interval, parametrized by $p \in (0, 1)$, all supoprted on the full interval, invariant under the doubling map and ergodic, so pairwise mutually singular ($\nu_{1/2}$ is Lebesgue). They are also self-similar and one can compute their Hausdorff dimension as follows:
$$\dim_H(\nu_p) = h_p/\log 2, \quad \text{where } h_p = -p \log p - (1 - p) \log(1 - p).$$
2. Krylov-Bogolyuboff Theorem (for a continuous map on a compact metric space, there is always an invariant Borel probability measure). (see [3, p.8])

References (these books are on 24-hour reserve at the Math Library):

1. William Parry, *Topics in ergodic theory*, Cambridge University Press, 1981.
2. Karl Petersen, *Ergodic Theory*, Cambridge University Press, 1989.
3. Mark Pollicott, *Lectures on Ergodic Theory and Pesin Theory on compact manifolds*, Cambridge University Press, 1993.
4. Peter Walters, *An Introduction to Ergodic Theory*, Springer, 1981.