

Lecture 10 (June 2) (See [2, Chapter 2] and [3, Section 6.2])

1. $f_n = I(\alpha|\alpha_1^n)$, $f^* = \sup f_n$. Maximal Lemma: $\mu\{f^* > \lambda\} \leq e^{-\lambda}$.
2. Corollary: $\int f^* d\mu \leq H(\alpha) + 1$.
3. $f_n \rightarrow f$ a.e. by the Martingale Convergence Theorem and in L^1 by the Corollary.
4. Shannon-McMillan-Breiman Theorem: $(n+1)^{-1}I(\alpha_0^n)$ converges a.e. to $h(T, \alpha)$ (assuming ergodicity; in general, it converges to the conditional expectation $E(f|\alpha_0^\infty)$).

Lecture 11 (June 4) (See [1, Chapter 4])

1. “Cookie-cutters” and conformal repellers.
2. Symbolic representation, cylinder sets.
3. Bounded distortion property and existence of the Lyapunov exponent.
4. Manning’s formula: dimension of an ergodic invariant measure is the entropy over the Lyapunov exponent (proof uses the Shannon-McMillan-Breiman Theorem and Birkhoff Ergodic Theorem).

Lecture 12 (June 6) (See [1, Chapter 5])

1. Definition and existence of pressure for a Hólder potential.
2. Existence of Gibbs measures.
3. Dimension of the cookie-cutter set is the zero of the pressure function $P(-s \log |f'|)$.

References (these books are on 24-hour reserve at the Math Library):

1. Kenneth Falconer, *Techniques in Fractal Geometry*, Wiley, 1997.
2. William Parry, *Topics in ergodic theory*, Cambridge University Press, 1981.
3. Karl Petersen, *Ergodic Theory*, Cambridge University Press, 1989.