

MATH 112 — SPRING 2009
WHAT TO EXPECT ON THE FINAL EXAM

- 6-8 questions (with multiple parts, of course) — The exam is about twice as long as a midterm, but you will have three hours to complete the exam.
- The exam is cumulative. So, you should expect to see problems from throughout the quarter on the exam. About half of the points on the exam will come from the newest material (Worksheets 19–24, the material on areas and integrals).
- I guarantee that you will have to take some derivatives using the derivative rules (WS 12, 13), evaluate some integrals (WS 23, 24), and answer questions about multi-variable functions and partial derivatives (WS 17).
- The most important thing to do to prepare for this exam is to practice doing problems. I suggest re-doing (or doing for the first time) all of the arrow problems from the text. Make sure that you know why you do each step — don't just flail about randomly until you get the answer. There should be a reason that you do everything you do.
- Do one or two old final exams so that you get an idea of their format. Remember that your exam will not look exactly like any old final exam. The way to ensure that you've studied everything is to study the problems from the text.
- Here's what you should consider while you prepare:

I. You should know **a lot** about derivatives. The derivative of a function gives us the slope of its tangent line. Slopes of tangent lines can be thought of as approximations of slopes of secant lines through two points on a graph that are very close together.

You should know what the graph of $f(x)$ tells you about the graph of $f'(x)$ (the derived graph of $f(x)$) and vice versa.

You should be very familiar with the process by which we first computed derivatives: computing and simplifying an expression like

$$\frac{f(m+h) - f(m)}{h}$$

until it was “safe” to let h go to 0.

You should recognize derivatives in our favorite contexts:

- the derivative of distance is instantaneous speed
- the derivative of total revenue is marginal revenue
- the derivative of total cost is marginal cost (as is the derivative of variable cost)
- the derivative of the amount of water in a vat is the rate of flow into the vat
- the derivative of altitude is rate of ascent

You should be able to take derivatives using the derivative rules.

II. You should be able to optimize functions. Remember that the most important tool in determining where a function is maximized or minimized is the derivative. Maxes and mins happen when a function has a horizontal tangent, i.e. where the

derivative is equal to 0. You should be very familiar with the different methods we have for finding a function's local and global optima: specifically, you should be able to apply our recipe for finding the global max and min of a function on an interval and you should know how to apply the Second Derivative Test on critical numbers to see if the critical number gives a local max or a local min.

You may want to review the concept of a demand curve from Worksheet 15 and, like I said before, you will see a problem about multi-variable functions and partial derivatives (Worksheet 17) on the exam.

Since most of you did the linear programming problem well on the second midterm, I will not test on this topic again. You're welcome.

III. You should know a bit about integrals. Know that $\int_a^b f(x) dx$ represents a "signed area" and is therefore a **number**. You may be asked to compute such an integral from the graph of $f(x)$ (in which case, you should count rectangles or compute the area of one or more trapezoids or other familiar shapes). You may also be asked to compute such an integral using the Fundamental Theorem of Calculus (the amazing bridge between the ideas of area and anti-derivatives).

On the other hand, $\int f(x) dx$ represents the most general anti-derivative of $f(x)$ and is therefore a function (with a "+K" on the end).

You should know about the function $A(m) = \int_0^m f(x) dx$ and its relationship to f (specifically, A is an anti-derivative of f , which means that f is the derivative of A). Remember that the graph of A is the accumulated graph of f and the graph of A always goes through the origin.

Finally, you should be familiar with integrals in our favorite contexts

- $\int_a^b MR(q) dq = TR(b) - TR(a)$, which is the change in TR from $q = a$ to $q = b$
- $\int_a^b MC(q) dq = TC(b) - TC(a) = VC(b) - VC(a)$, which is the change in TC (or VC) from $q = a$ to $q = b$
- $\int_a^b \text{speed } dt = \text{distance traveled from } t = a \text{ to } t = b$
- $\int_a^b \text{rate of flow } dt = \text{amount that flows in from } t = a \text{ to } t = b$