

MATH 112 - Spring 2005
Exam II - Version 1
Hints and Answers

1. (4 points each)

(a) $\frac{dy}{dx} = \frac{1}{x^{1/3} + e^{4x^5}} \cdot \left(\frac{1}{3}x^{-2/3} + e^{4x^5} \cdot 20x^4 \right)$

(b) $G'(t) = 10 \left[\frac{x^3}{1 + e^{-8x}} \right]^9 \cdot \left[\frac{(1 + e^{-8x})(3x^2) - x^3(e^{-8x})(-8)}{(1 + e^{-8x})^2} \right]$

(c) $\frac{\partial z}{\partial y} = -\frac{1}{2} [x^2 + (4y + 3)^2]^{-3/2} \cdot 2(4y + 3)(4)$

(d) $f_x(x, y) = y[x^2 \cdot e^x + e^x \cdot 2x]$

2. (a) (4 points) Draw the vertical line $x = 20$, the horizontal line $y = 8$, and the line $5x + 10y = 133$ (which has intercepts $(0, 13.3)$ and $(26.6, 0)$). The feasible region is a five-sided polygon.

(b) (5 points) The feasible region has five vertices: $(0, 0)$, $(0, 8)$, $(20, 0)$, $(10.6, 8)$ and $(20, 3.3)$.

(c) (3 points) The optimum values of the objective function must occur at a vertex of the feasible region. Plug all of your vertices into the objective function and choose the largest and smallest values.

ANSWER: minimum = 0; maximum = 3.06

3. (a) (5 points) HINT: Find the profit function: $P(q) = -0.1q^3 - 2.4q^2 + 86.4q - 330$. Take the derivative: $P'(q) = -0.3q^2 - 4.8q + 86.4$. Set $P'(q) = 0$ and solve for q , using the quadratic formula. You get one positive solution: $q = 10.76$. To apply the Second Derivative Test, take $P''(q)$: $P''(q) = -0.6q - 4.8$. Since $P''(10.76)$ is negative, profit has a local maximum at $q = 10.76$.

ANSWER: $q = 10.76$ Items

(b) (5 points) HINT: $AVC(q) = 0.1q^2 - 0.6q + 3.6$. $AVC'(q) = 0.2q - 0.6$. Set $AVC'(q) = 0$ and solve for q : $q = 3$. Compute $AVC(1)$, $AVC(3)$, and $AVC(10)$.

ANSWER: smallest = \$2.70 per Item; largest = \$7.60 per Item

(c) (4 points) HINT: Marginal cost is the derivative of total cost: $MC(q) = 0.3q^2 - 1.2q + 3.6$. Find $MC'(q)$ and $MC''(q)$: $MC'(q) = 0.6q - 1.2$ and $MC''(q) = 0.6$. So, $MC''(1) = 0.6$. Since $MC''(1)$ is positive, MC is concave up at $q = 1$.

ANSWER: concave up

4. (a) (4 points) HINT: Set the partial derivatives equal to 0 and solve the resulting system of equations.

ANSWER: $y = 1.525x + 1.5625$

(b) (2 points) HINT: $\frac{E(1, 2.003) - E(1, 2)}{0.003} \approx E_m(1, 2)$

ANSWER: 59

(c) (2 points) HINT: You need $\frac{\partial E}{\partial b}$ to be positive. Choose any pair of numbers m and b that make $2b + 15m - 26 > 0$.