MATH 112 - Spring 2005
Exam II - Version 1
Hints and Answers

1. (4 points each)

   (a) $\dfrac{dy}{dx} = \dfrac{1}{x^{1/4} + e^{6x^3}} \cdot \left(\dfrac{1}{4}x^{-3/4} + e^{6x^3} \cdot 18x^3\right)$

   (b) $G'(t) = 14\left[\dfrac{x^5}{1+e^{-12x}}\right]^{13} \cdot \left[\dfrac{(1+e^{-12x})(5x^4) - x^5(e^{-12x})(-12)}{(1+e^{-12x})^2}\right]$

   (c) $\dfrac{\partial z}{\partial y} = -\dfrac{1}{2}\left[x^2 + (5y+1)^2\right]^{-3/2} \cdot 2(5y+1)(5)$

   (d) $f_x(x,y) = y[x^2 \cdot e^x + e^x \cdot 2x]$

2. (a) (4 points) Draw the vertical line $x = 18$, the horizontal line $y = 10$, and the line $5x + 10y = 133$ (which has intercepts $(0, 13.3)$ and $(26.6, 0)$). The feasible region is a five-sided polygon.

   (b) (5 points) The feasible region has five vertices: $(0,0)$, $(0,10)$, $(18,0)$, $(6.6,10)$ and $(18,4.3)$.

   (c) (3 points) The optimum values of the objective function must occur at a vertex of the feasible region. Plug all of your vertices into the objective function and choose the largest and smallest values.
   ANSWER: minimum = 0; maximum = 7.55

3. (a) (5 points) HINT: Find the profit function: $P(q) = -0.1q^3 - 2.4q^2 + 86.4q - 330$. Take the derivative: $P'(q) = -0.3q^2 - 4.8q + 86.4$. Set $P'(q) = 0$ and solve for $q$, using the quadratic formula. You get one positive solution: $q = 10.76$. To apply the Second Derivative Test, take $P''(q)$: $P''(q) = -0.6q - 4.8$. Since $P''(10.76)$ is negative, profit has a local maximum at $q = 10.76$.
   ANSWER: $q = 10.76$ Items

   (b) (5 points) HINT: $AVC(q) = 0.1q^2 - 0.6q + 3.6$. $AVC'(q) = 0.2q - 0.6$. Set $AVC'(q) = 0$ and solve for $q$: $q = 3$. Compute $AVC(1)$, $AVC(3)$, and $AVC(10)$.
   ANSWER: smallest = \$2.70 per Item; largest = \$7.60 per Item

   (c) (4 points) HINT: Marginal cost is the derivative of total cost: $MC(q) = 0.3q^2 - 1.2q + 3.6$. Find $MC'(q)$ and $MC''(q)$: $MC'(q) = 0.6q - 1.2$ and $MC''(q) = 0.6$. So, $MC''(1) = 0.6$. Since $MC''(1)$ is positive, $MC$ is concave up at $q = 1$.
   ANSWER: concave up

4. (a) (4 points) HINT: Set the partial derivatives equal to 0 and solve the resulting system of equations.
   ANSWER: $y = 1.525x + 1.5625$

   (b) (2 points) HINT: $\dfrac{E(4, 1.002) - E(4,1)}{0.002} \approx E_m(4,1)$
   ANSWER: -38

   (c) (2 points) HINT: You need $\frac{\partial E}{\partial b}$ to be positive. Choose any pair of numbers $m$ and $b$ that make $2b + 15m - 26 > 0$.