MATH 112
REVIEW FOR EXAM II
OPTIMIZATION

I. Derivative Rules

- There will be a page of derivatives on the exam. Know how to apply all the derivative rules. (WS 12 and 13)

II. Functions of One Variable

- Be able to distinguish between local and global optima.
- Be able to find the global maximum and minimum of a function $y = f(x)$ on the interval from $x = a$ to $x = b$, using the fact that optima may only occur where $f(x)$ has a horizontal tangent line and at the endpoints of the interval.

  Step 1: Compute $f'(x)$.

  Step 2: Find all values of x at which $f'(x) = 0$.

  Step 3: Plug all the values of x from Step 2 *that are in the interval from a to b* and the endpoints of the interval into the function $f(x)$.

  Step 4: Sketch a rough graph of $f(x)$ and pick off the global max and min.
- Be familiar with the following two applications:
  - Maximizing $TR(q)$ starting with a demand curve. (WS 15)
  - Optimizing the slope of a diagonal line through a given curve. (WS 16)
- Understand how to use the Second Derivative Test. (WS 17)

  If $f'(a) = 0$ **and**:
  - $f''(a) > 0$, then $f(x)$ has a local min at $x = a$.
  - $f''(a) < 0$, then $f(x)$ has a local max at $x = a$.
  - $f''(a) = 0$, then the test tells you nothing.

  IMPORTANT!!! For the Second Derivative Test to work, you must have $f'(a) = 0$. If $f''(a) > 0$ but $f'(a) \neq 0$, then the graph of $f(x)$ is concave up at $x = a$ but $f(x)$ does not have a local min there.

III. Functions of Two Variables

- Be able to compute overall, incremental, and instantaneous rates of change of a function of two variables. (WS 18A)
- Be able to compute partial derivatives using all the derivative rules.
- Know how to find the candidates for maxima and minima in a function of two variables. (Take both partial derivatives, set them equal to 0, and solve the resulting system of equations.)

- Know the procedure for finding the best-fitting line for a set of data. (WS 18)

  Step 1: Given n points (x_i, y_i), compute $\sum x_i$, $\sum y_i$, $\sum x_i^2$, $\sum y_i^2$, $\sum x_i y_i$.

  Step 2: Use the sums from Step 1 to find the formula for the mean squared error function $E(b, m)$.

  Step 3: Compute $\frac{\partial E}{\partial b}$ and $\frac{\partial E}{\partial m}$.

  Step 4: Solve the system of equations $\frac{\partial E}{\partial b} = 0$ and $\frac{\partial E}{\partial m} = 0$ for m and b. These are the slope and y-intercept of the best-fitting line $y = mx + b$.

- Know when it's appropriate to take logs to make exponential data look linear. (WS 14)

- Be able to convert a linear model for $\ln y$ into an exponential model for y.

- Be able to solve a linear programming problem. (WS 19)

  Step 1: Find the objective function.

  Step 2: Find the constraints.

  Step 3: Graph the feasible region and find its vertices.

  Step 4: Plug **all** vertices into the objective function. (The max and min of the objective function must occur at one of the vertices.)