Review Problems for Exam I
MATH 126 – Autumn 2011

Here are some problems from old exams that will help you prepare for the midterm. This is not intended to be an exhaustive review of the material. You will be expected to understand how to do all the assigned homework. I suggest you work these problems only after re-working most or all of the homework.

1. Find all values of x such that $\langle 4, 5, x\rangle$ and $\langle 3x, 7, x\rangle$ are orthogonal.
2. Find a vector that is orthogonal to the vector $\langle 11, 3, -5\rangle$ and has length 7.
3. The set of points that are twice as far from the origin as they are from the point $(5, 5, 5)$ is a sphere. Find its center and radius.
4. Find the parametric equations for the line that is the intersection of the plane $x + y + 3z = 8$ with the plane $2x - y - z = 4$.
5. Find the equation of the plane that passes through the point $(3, -1, 2)$ and contains the line
$$x = 5 - t, y = 3 + 3t, z = 8 + 2t.$$
6. Find the equation of the plane containing the line of intersection of the two planes
$$x + y + z + 5 = 0 \text{ and } 3x + 2y - z + 2 = 0$$
and the point $(1, 2, 1)$.
7. Find the equation of the plane through the three points $(-3, 4, 0)$, $(1, 7, -3)$ and $(2, -5, 3)$.
8. Find the point of intersection of the two lines
$$x = 4 - t, y = 6 + 2t, z = -1 + 3t \text{ and } x = 1 + 2t, y = 14 - 8t, z = 7 - 4t.$$
9. Let S be the surface defined as the set of points p (in three-dimensional space) such that the distance from p to the plane $y = 5$ equals the distance from p to the line
$$x = t, y = 1, z = 2.$$
   (a) Find an equation for S.
   (b) Find the equation of the trace of S in the plane $z = 6$. Describe the trace (i.e. what kind of curve is it?).
10. Consider the curve defined parametrically by the parametric equations
$$x = \ln\ln t, y = \ln t - (\ln t)^2.$$
Find the equation of the tangent line to the curve at the point $t = e$.
11. Find the length of the curve
$$x = 4\sqrt{t},\ y = \frac{t^3}{3} + \frac{1}{2t^2},\ 1 \le t \le 4.$$
12. Consider the curve defined by the vector equation
$$\vec{r}(t) = \langle 4t, 5t^3, 2t^2\rangle$$
   (a) Find the unit tangent vector $\vec{T}(t)$ at the point where $t = 1$.
   (b) Find the parametric equations of the tangent line the curve at the point where $t = 1$.
13. Consider the space curve defined by the vector function
$$\mathbf{r}(t) = \langle t^2 - 3, 2e^{t-6}, 4 - t^3\rangle.$$
Find all values of t such that a tangent vector to $\mathbf{r}(t)$ is parallel to the line
$$x = 3 + 4t, y = \frac{4}{3} + \frac{2}{3}t, z = -36t.$$

14. Find the slope of the tangent line to the polar curve

$$r = \frac{1}{\theta}, \theta > 0$$

at the point where it intersects the cartesian curve

$$x^2 + y^2 = \frac{1}{9}.$$

15. Find all points of intersection between the curve defined by the polar equation

$$r = 4 \csc \theta$$

and the line $y = x$.

16. Consider the curve in the xy-plane defined by the position vector function

$$\overrightarrow{r}(t) = \langle t^2 - 3t, t^2 + 2t \rangle$$

Find the t-value of the point of maximum curvature on this curve.

17. For any $m > 0$, the helix determined by the position function

$$\vec{r}(t) = \langle \cos t, \sin t, mt \rangle$$

has constant curvature that depends on m. Find the value of m such that the radius of curvature at any point on the curve is 3.