MATH 126
Exam I Review - Hints and Answers

1. We need all x such that the dot product of the two vectors is equal to 0:

$$\langle 4,5,x\rangle \cdot \langle 3x,7,x\rangle = 12x+35+x^2 = (x+7)(x+5).$$

This is 0 for $x=-7$ and $x=-5$.

2. Many (in fact, infinitely many) correct answers to this one. Here is one.
$\langle 0, \frac{35}{\sqrt{34}}, \frac{21}{\sqrt{34}}\rangle$.

3. Suppose $P(x,y,z)$ is a point on the sphere. Let A be the point $(5,5,5)$ and O be the origin. Then the distance from P to O is twice the distance from P to A:

$$\sqrt{(x-0)^2+(y-0)^2+(z-0)^2} = 2\sqrt{(x-5)^2+(y-5)^2+(z-5)^2}.$$

Square both sides and complete the square(s) to get the standard form of the equation for the sphere. Read off the center and radius.
ANSWER: center $\left(\frac{20}{3},\frac{20}{3},\frac{20}{3}\right)$, radius $\frac{10}{\sqrt{3}}$.

4. The normal vectors for the planes are $\vec{n_1}=\langle 1,1,3\rangle$ and $\vec{n_2}=\langle 2,-1,-1\rangle$. So, a direction vector for the line is $\vec{v}=\vec{n_1}\times\vec{n_2}=\langle 2,7,-3\rangle$. To find a point on the line, you could find where the line intersects the xy-plane: $(4,4,0)$. Then, one set of parametric equations for the line is: $x=4+2t, y=4+7t, z=-3t$. Other correct answers are possible.

5. ANSWER: $x+y-z=0$

6. ANSWER: $19x+10y-17z=22$

7. ANSWER: $6x+9y+17z=18$

8. ANSWER: $(2,10,5)$

9. (a) HINTS: Let $P(x,y,z)$ be a point on the surface. The distance from P to the plane $y=5$ is $|y-5|$. (Why?) The distance from P to the line $x=t,\ y=1,\ z=2$ is $\sqrt{(y-1)^2+(z-2)^2}$. (Why?)
ANSWER: An equation for the surface S is $|y-5|=\sqrt{(y-1)^2+(z-2)^2}$. Square both sides and simplify: $y=-\frac{1}{8}(z-2)^2+3$.
(b) ANSWER: $y=1$, a line

10. ANSWER: $y=-x$

11.
$$\frac{dx}{dt}=\frac{2}{\sqrt{t}} \text{ and } \frac{dy}{dt}=t^2-\frac{1}{t^3}$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}=\sqrt{\frac{4}{t}+t^4-\frac{2}{t}+\frac{1}{t^6}}=\sqrt{t^4+\frac{2}{t}+\frac{1}{t^6}}=\sqrt{\left(t^2+\frac{1}{t^3}\right)^2}=t^2+\frac{1}{t^3}.$$

The length of the curve is:

$$L=\int_1^4 t^2+\frac{1}{t^3}\,dt=\frac{1}{3}t^3-\frac{1}{2t^2}\Big|_1^4=\ldots=\frac{687}{32}$$

12. (a) $\vec{T}(1)=\left\langle \frac{4}{\sqrt{257}},\frac{15}{\sqrt{257}},\frac{4}{\sqrt{257}}\right\rangle$; (b) $x=4+4t, y=5+15t, z=2+4t$

13. ANSWER: $t=6$

14. ANSWER: The slope of the tangent line is

$$\frac{dy}{dx} = \frac{3\cos 3 - \sin 3}{-(\cos 3 + 3\sin 3)}$$

15. Since $y = r\sin\theta$, $\csc\theta = \frac{r}{y}$. Note that the curve $r = 4\csc\theta$ does not go through the origin since $\csc\theta$ is never equal to 0. This means that r is never 0. So,

$$r = 4\csc\theta \Rightarrow r = \frac{4r}{y} \Rightarrow y = 4.$$

That is, the polar curve $r = 4\csc\theta$ is the horizontal line $y = 4$ in the Cartesian plane. The intersection of this line with the line $y = x$ is the point with Cartesian coordinates $(4, 4)$ and polar coordinates $(4\sqrt{2}, \frac{\pi}{4})$.

16. HINT: The curvature is

$$\kappa(t) = \frac{10}{(8t^2 - 4t + 13)^{3/2}}.$$

This is largest when the denominator is smallest.

ANSWER: $t = \dfrac{1}{4}$

17. ANSWER: $m = \sqrt{2}$