

## Hot Topics for the Math 126 Final Exam

1. Be able to deal with vectors.
  - (a) operations on and between (sums, products, magnitudes)
  - (b) geometry (angles between, parallel, orthogonal)
2. Be able to find equations of lines and planes. (Lots of possible questions here — see the study guide for midterm 1.)
3. Be able to deal with parametric curves/vector functions.

Specifically: derivatives ( $\vec{r}'(t)$  and  $\frac{dy}{dx}$ ), tangent lines, arc lengths, unit tangent and normal vectors, curvature, velocity and acceleration, normal planes.

4. Be able to deal with multi-variable functions.

Specifically: level curves, partial derivatives, critical points, tangent planes, linear approximation, optimization.
5. Be able to deal with double integrals.

Specifically: volumes and areas, in polar coordinates, changing the order of integration, center of mass.

6. Taylor polynomials, Taylor's inequality, and Taylor series!!!

- (a) Given a function  $f(x)$  and a base  $b$ , be able to:
  - i. find the  $n^{\text{th}}$  Taylor polynomial for  $f(x)$  based at  $b$ :

$$T_n(x) = \sum_{k=0}^n \frac{f^{(k)}(b)}{k!} (x - b)^k.$$

- ii. find the Taylor series for  $f(x)$  based at  $b$ :

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(b)}{k!} (x - b)^k.$$

- iii. give an upper bound for  $|f(x) - T_n(x)|$  on an interval  $I$  using Taylor's inequality

$$|f(x) - T_n(x)| \leq \frac{M}{(n+1)!} |x - b|^{n+1},$$

where  $|f^{(n+1)}(x)| \leq M$  for all  $x$  in  $I$ .

- iv. find an interval  $J$  on which  $|f(x) - T_n(x)|$  is less than some specified error for some specific value of  $n$ .
  - v. find a value of  $n$  for which  $|f(x) - T_n(x)|$  is less than some specified error for all  $x$  in an interval  $I$ .
- (b) Know the Taylor series for  $e^x$ ,  $\sin x$ ,  $\cos x$ , and  $\frac{1}{1-x}$  based at  $b = 0$  (including the intervals on which they converge) and be able to use appropriate substitutions, differentiation, and integration to find related Taylor series (including the intervals on which they converge).