MATH 145
WINTER 2008
SAMPLE EXAM I — ANSWERS

1. (a) $\dfrac{dy}{dx} = \dfrac{1}{x^3 e^{-x}}[x^3(-e^{-x}) + e^{-x}(3x^2)]$

   (b) $\dfrac{dy}{dx} = x^2 \cdot \dfrac{1}{1+5e^x} \cdot 5e^x + 2x\ln(1+5e^x)$

   (c) $\dfrac{dy}{dx} = \dfrac{(1+x)e^x - e^x}{(1+x)^2}$

   (d) $\dfrac{dy}{dx} = \dfrac{(x^2+5x)\cdot\frac{1}{2x+7}\cdot 2 - [\ln(2x+7)]\cdot(2x+5)}{(x^2+5x)^2}$

   (e) $\dfrac{dy}{dx} = \dfrac{1}{2}(2x+6)^{-1/2}\cdot 2 = \dfrac{1}{\sqrt{2x+6}}$

   (f) $\dfrac{dy}{dt} = (3t^2+7)^4 \cdot 7(t^3-t)^6(3t^2-1) + (t^3-t)^7 \cdot 4(3t^2+7)^3(6t)$

   (g) $\dfrac{dy}{dx} = 7\left[x-(x+3)^4\right]^6\left[1-4(x+3)^3\right]$

   (h) $\dfrac{dy}{dx} = e^{5x}\cdot 2(x^3+4)(3x^2) + (x^3+4)^2\cdot e^{5x}\cdot 5$

2. (a) The maximum value is -1.

   (b) The maximum value is $\frac{1}{3}$.

   (c) $g(x)$ has no critical points since $g'(x) = f''(x)$ is never 0.

3. The PRINTED MATTER will have maximal area when $x = \sqrt{500}$ and $y = 2\sqrt{500}$.

4. (a) $f'(x) = -2xe^{-x^2}$ and $f''(x) = 2e^{-x^2}(2x^2-1)$ (simplified)

   (b) $f'(x) = 0$ only at $x = 0$.

   (c) $f(x)$ is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. $f(x)$ is concave up on $\left(-\infty, -\sqrt{\frac{1}{2}}\right)$ and on $\left(\sqrt{\frac{1}{2}}, \infty\right)$, concave down on $\left(-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}\right)$.

   (d) The y-intercept is $f(0) = 1$. Since $f(x)$ is never 0, there is no x-intercept.

   (e) The limits are both equal to 0. This means that the graph of $f(x)$ will approach the line $y = 0$ (the x-axis) as x approaches $\pm\infty$. That is, the x-axis is a horizontal asymptote for the graph of $f(x)$.

5. (a) -1

   (b) $-\frac{1}{6}$

   (c) -5

   (d) 5

   (e) 1