

**MATH 145**  
**WINTER 2008**  
**SAMPLE FINAL EXAM**

This is intended to give you an idea of the level of difficulty of the problems you can expect to see on the final exam. This is not an exhaustive review. You will be expected to understand all concepts covered in class and on homework.

On problems involving the normal distribution, you may use the table of values handed out in class.

1. Compute the derivative. Do not simplify.

(a)  $y = \sin(\cos(\ln x))$

(b)  $y = \sqrt{\frac{\sin x}{xe^{3x-5}}}$

(c)  $y = \frac{\tan^3 x}{x^2 + 5}$

(d)  $y = \sin(x\sqrt{x+1})$

(e)  $y = \frac{e^{\tan x}}{x^3 + 2x}$

(f)  $y = \sin(x^2) \cdot \cos\left(\frac{1}{x^2}\right)$

2. Let  $f(x) = e^{-x^2}$ .

(a) Compute  $f'(x)$  and  $f''(x)$ .

(b) Find all critical points of  $f(x)$ . At each one, state whether  $f(x)$  has a relative maximum, a relative minimum, or a horizontal inflection point (i.e., a “seat”).

(c) Find all inflection points of  $f(x)$ .

(d) Find the  $x$ - and  $y$ -intercepts of  $f(x)$ .

(e) Does  $f(x)$  have any asymptotes? If so, what are they?

(f) Sketch a rough graph of  $f(x)$ , clearly labeling all intercepts, critical points, and inflection points.

3. Evaluate the integrals.

(a)  $\int \frac{\ln 4x}{2x} dx$

(b)  $\int \ln(2x + 4) dx$

(c)  $\int_0^{\pi/5} x \cos 5x dx$

(d)  $\int_0^4 (x - 3)^2 dx$

(e)  $\int \ln(3x + 5) dx$

(f)  $\int \frac{3 - 2x}{\sqrt{x^2 - 3x + 1}} dx$

- (g)  $\int 4x^5 \sec^2(x^6) dx$
- (h)  $\int_0^{\pi/2} \frac{\sin x}{1 + \cos x} dx$
- (i)  $\int \frac{\cos x}{\sin^2 x(1 - \sin x)} dx$

4. For each of the following, evaluate the improper integral or show that it diverges.

- (a)  $\int_0^4 \frac{dx}{4 - x}$
- (b)  $\int_0^{\infty} 4e^{-4x} dx.$
- (c)  $\int_{-\infty}^{\infty} e^{-|x|} dx$

5. A fence is to be built to enclose a rectangular pasture whose area is 15,625 square meters. One side of the pasture will be bounded by a river, so no fencing is needed there. Fencing for the side opposite the river costs \$4 per meter, while fencing for the other two sides costs \$2 per meter. Find the cost of the least expensive fence. (As part of your work, you must show that your answer does indeed give a minimum value of cost.)

6. A box is to be constructed so that its top and base are squares. The cost of the material for the bottom of the box is \$0.10 per square inch; for the top, the cost is \$0.08 per square inch; and for the sides, \$0.20 per square inch. If the box must hold 320 cubic inches, what dimensions will minimize the total cost to construct the box?

7. The length (in inches) of a fish caught at random in a lake is a continuous random variable  $X$  that is normally distributed with mean  $\mu = 13$  and standard deviation  $\sigma = 3$ .

- (a) What percentage of fish are at least 6 inches long?
- (b) What is the probability that a fish is no more than 8 inches long?
- (c) What percentage of fish are between 13 and 15 inches long?

8. Suppose  $X$  is a continuous random variable that is exponentially distributed with parameter  $\lambda = 4$ .

- (a) Show that  $EX = \frac{1}{4}$ .
- (b) Find  $P(0 \leq X \leq 5)$ .

9. The function

$$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x^2 e^{-\frac{1}{4}x^3} & \text{for } 0 \leq x \leq b \\ 0 & \text{for } x > b \end{cases}$$

is a probability density function of a continuous random variable. What is  $b$ ?

10. Suppose  $X$  is a continuous random variable that is normally distributed with  $\mu = 3$  and  $P(3 \leq X \leq 5) = 0.4332$ . What is the standard deviation  $\sigma$  of this normal distribution?

11. A machine is designed to produce screws that are 2.5 cm long. However, no machine is perfect. The actual length of a screw produced by this machine is a continuous random variable that has a normal distribution with mean  $\mu = 2.5$  cm and standard deviation  $\sigma = 0.2$  cm.

- (a) What percentage of the screws produced by this machine are longer than 2.7 cm?
  - (b) What percentage of screws produced by this machine are between 2.4 and 2.75 cm long?
12. The *median* of continuous random variable  $X$  is defined to be the number  $m$  with the property that

$$P(X \leq m) = \frac{1}{2}.$$

Find the median of each of the following distributions.

- (a) the standard normal distribution (Draw a picture to justify your answer, but you need not show any more work than that.)
- (b) the uniform distribution:  $f(x) = \frac{1}{b-a}$  on the interval  $[a, b]$  and  $f(x) = 0$  everywhere else
- (c) the exponential distribution:  $f(x) = \lambda e^{-\lambda x}$  on the interval  $[0, \infty)$  ( $\lambda$  is a positive constant) and  $f(x) = 0$  for  $x < 0$