MATH 300 B
Final Exam
March 17, 2011

Name ____

Student ID #_________________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:________________________________

1	20	
2	10	
3	10	
4	5	
5	10	
6	10	
Total	65	

- Your exam should consist of this cover sheet, followed by 6 problems. Check that you have a complete exam.
- You are not allowed to use any outside sources on this exam.
- Turn your cell phone OFF and put it AWAY for the duration of the exam.

GOOD LUCK!

1. (20 points) For each of the following, determine whether the statement is TRUE or FALSE. You do not need to provide any justification.

   (a) The function $f : \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ defined by $f(x) = (x - 3, 2x + 1)$ is a bijection.

   (a) TRUE______FALSE ______

   (b) The function $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined by $g(n, m) = (m - 3, 2n + 1)$ is a bijection.

   (b) TRUE______FALSE ______

   (c) There exists a bijection $f : \mathbb{Z} \to \mathbb{N} \times \mathbb{N}$.

   (c) TRUE______FALSE ______

   (d) If A and B are countable, then $A - B$ is countable.

   (d) TRUE______FALSE ______

   (e) $\mathbb{Q} - \mathbb{Z}$ is denumerable.

   (e) TRUE______FALSE ______

   (f) Let $S = \{(x, y) \in \mathbb{N} \times \mathbb{R} : xy = 1\}$. Then S is uncountable.

   (f) TRUE______FALSE ______

   (g) Suppose $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ and define $h : \mathbb{R} \to \mathbb{R}$ by $h(x) = f(x) + g(x)$ for all $x \in \mathbb{R}$. If f and g are onto, then h must be onto.

   (g) TRUE______FALSE ______

   (h) Suppose $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$. If g is decreasing, then $g \circ f$ must also be decreasing.

   (h) TRUE______FALSE ______

   (i) If $f : A \to B$, $g : B \to C$, and $g \circ f : A \to C$ is onto, then f must be onto.

   (i) TRUE______FALSE ______

   (j) If $f : A \to B$, $g : B \to C$, and $g \circ f : A \to C$ is onto, then g must be onto.

   (j) TRUE______FALSE ______

2. (10 points) Prove that $3|(2^{2n}-1)$ for every $n \in \mathbb{N}$.

3. (10 points)

   (a) Define a relation R on $\mathbb{Z}$ by

   $$R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \text{ is even}\}.$$

   Is R an equivalence relation? (Justify your answer, of course.)

   (b) Let A be a non-empty set and T be a relation on A. Prove or give a counterexample of the following statement: If T is symmetric and transitive and the domain of T is A, then T is an equivalence relation.

4. (5 points) Let A, B, and C be sets and suppose $f : A \to B$, $g : B \to C$, and $h : B \to C$. Prove that, if f is onto and $g \circ f = h \circ f$, then $g = h$.

5. (10 points) Suppose A and B are sets, $f : A \to B$, and $C \subseteq B$.

   (a) Prove that
   $$A - f^{-1}(C) \subseteq f^{-1}(B - C).$$

   (b) Suppose A is countable. Prove that, if B is uncountable, then $B - A$ is uncountable.

6. (10 points) Define $f : \mathbb{R} - \{1\} \to \mathbb{R} - \{3\}$ by

$$f(x) = \frac{3x}{x-1}.$$

Is f a bijection? Prove your answer.