MATH 300 D
Winter 2011
Final Exam Hints and Answers

1. (a) T; (b) F; (c) T; (d) T*; (e) T; (f) F; (g) F; (h) F; (i) F; (j) T

   *NOTE: (d) is true using the book's definition of the word *countable* (finite or denumerable). If you interpret the word *countable* to mean *countably infinite* (denumerable), then this statement is FALSE: if A and B are countably infinite, then $A - B$ may be finite.

2. HINT: Use induction on n.

3. (a) R is an equivalence relation.

   (b) HINT: Make sure that you explicitly use the hypothesis that $\text{Dom}(T) = A$.

4. HINT: First, verify that g and h have the same domain (B) and then show that, for all $y \in B$, $g(y) = h(y)$.

5. (a) HINT: Choose an arbitrary $x \in A - f^{-1}(C)$ and show that x must be an element of $f^{-1}(B - C)$.

   (b) HINT: You are given that A is countable. Use contraposition: suppose $B - A$ is countable and show that this means that B is countable. You may want to use the facts that $(B - A) \cup A = A \cup B$ and $B \subseteq A \cup B$.

6. HINT: Showing that f is one-to-one is straight-forward. To prove f is onto, choose an arbitrary $y \in \mathbb{R} - \{3\}$ and let $x = \frac{y}{y-3}$. It's easy to show that then $f(x) = y$. But you must also show that $x \in \mathbb{R} - \{1\}$. Since $y \neq 3$, consider the two cases: if $y > 3$, you can show that $x > 1$ and, if $y < 3$, you can show that $x < 1$.