Math 524

Homework due 10/18/00

Problem 1. Let (X, ρ) be a metric space. Let $E \subset X$.

1.1 Prove that every uniformly convergent sequence of bounded functions on E is uniformly bounded on E.

1.2 Assume that $\{f_n\}$ and $\{g_n\}$ are sequences of bounded functions which converge uniformly on E. Prove that $\{f_n g_n\}$ converges uniformly on E.

1.3 Assume that $\{f_n\}$ and $\{g_n\}$ converge uniformly on E. Does $\{f_n g_n\}$ converge uniformly on E?

Problem 2. Let (X, ρ) be a metric space. Let $K \subset X$ be compact. For each $k \in \mathbb{N}$, let $f_k : K \to \mathbb{R}$ be continuous and satisfy:

(i) $f_k(x) \geq 0$ for $x \in K$.

(ii) $f_k(x) \to 0$ pointwise in K.

(i) $f_k(x) \leq f_\ell(x)$ for $x \in K$ whenever $k \geq \ell$.

Prove that $f_k(x) \to 0$ uniformly on K.

Problems from Royden:

Chapter 7, Section 8: problems 36(a), 43.
Chapter 7, Section 10: problems 47, 50.