

Math 524

Homework due 11/15/00

Problem 1. (Prelim) Let $\{f_n\}_{n \geq 1}$ be a sequence of non-negative functions defined on a measure space (X, \mathcal{M}, μ) , such that for every $n \geq 1$,

$$\int_X f_n d\mu \leq 1.$$

Prove that

$$\limsup_{n \rightarrow \infty} (f_n(x))^{1/n} \leq 1 \quad \mu - \text{a.e. } x \in X.$$

Problem 2. (Prelim) Show, with justification of each step, that

$$\int_0^1 \left(\sum_{n=1}^{\infty} x^n \frac{\cos(2^n \pi x)}{n} \right) dx = \sum_{n=1}^{\infty} \left(\int_0^1 x^n \frac{\cos(2^n \pi x)}{n} dx \right).$$

Problems from Folland:

Chapter 2, Section 3: problems 18, 19, 20, 21, 27, 28 (a-c).