Math 524

Homework due 11/22/00

$*$ **Problem:** Let $(X, \mathcal{M}, \mu)$ be a measure space. Assume $\mu(X) < \infty$. Let $\{f_n\}_{n\geq 1}$ be a sequence in $L^1(\mu)$ satisfying:

1. $f_n \to f$ $\mu - a.e$ in X

2. Given $\epsilon > 0$ there is $\delta > 0$ so that

$$\sup_{n\geq 1} \int_E |f_n|\, d\mu < \epsilon, \quad \text{whenever} \quad \mu(E) < \delta.$$

Show that $f \in L^1(\mu)$ and that $f_n \to f$ in $L^1(\mu)$, i.e.

$$\lim_{n\to\infty} \int |f_n - f|\, d\mu = 0.$$

Problems from Folland:

Chapter 2, Section 3: problem 23.

Chapter 2, Section 4: problems 33, 34, 36, 40, 41, 44.

Hint for problem 36: see problems 3 and 5 in Chapter 2, Section 1.