

January 28, 2008

Problem 2.33

Three children are in line. From a collection of 2 red hats and 3 black hats, the teacher places a hat on each child's head. The third child sees the hats on the first 2 heads, the middle child sees the hat on the first head, and the first child sees no hats. The children, who reason carefully, are told to speak out as soon as they can determine the color of the hat they are wearing. After 30 seconds, the front child correctly names the color of her hat. Which color is it, and why?

Problem 2.25

For $a \in \mathbb{R}$ and $f : \mathbb{R} \rightarrow \mathbb{R}$, show that (1) and (2) have different meanings.

1. $(\forall \epsilon > 0)(\exists \delta > 0)(\forall x \in \mathbb{R})$
 $[|x - a| < \delta] \Rightarrow [|f(x) - f(a)| < \epsilon].$

2. $(\exists \delta > 0)(\forall \epsilon > 0)(\forall x \in \mathbb{R})$
 $[|x - a| < \delta] \Rightarrow [|f(x) - f(a)| < \epsilon].$

Problem 2.34

For each statement below about the natural numbers, decide whether it is true or false, and prove your claim using only properties of the natural numbers.

1. If $n \in \mathbb{N}$ and $n^2 + (n + 1)^2 = (n + 2)^2$, then $n = 3$.
2. For all $n \in \mathbb{N}$, it is false that $(n - 1)^3 + n^3 = (n + 1)^3$.