February 13, 2008

Midterm - Friday February 15, 2008!

Problem 3.53

Suppose that F is a polynomial of degree n defined by

$$F(x) = \sum_{i=0}^{n} c_i x^i$$

and has zeros $\alpha_1, \alpha_2, \cdots, \alpha_n$ such that $\alpha_i \neq 0$ for all i. Derive a formula for

$$\sum_{i=1}^{n} \frac{1}{\alpha_i}$$

in terms of $c_0, c_1, \cdots, c_n$.

Hint: First show that

$$F(x) = c_n \prod_{i=1}^{n} (x - \alpha_i).$$

The Fibonacci numbers are defined by the recurrence relation $f_1 = 1$, $f_2 = 1$ and

$$f_n = f_{n-1} + f_{n-2} \quad \text{for} \quad n \geq 3.$$

Prove that for $n \geq 1$, f_{3n} is even.