Math 310

Homework due 03/12/08

Reading: Preface for the student. Chapter 1. Chapter 2. Chapter 3: The principle of induction, Strong induction. Chapter 4: Bijections. Injections and Surjections. Composition of functions. Cardinality.

Problem 1. Let X be an uncountable set, Y be a countable set, and $f : X \to Y$. Prove that some element of Y has an uncountable pre-image. That is there exists $y \in Y$ such that

$$\{x \in X : f(x) = y\} \quad \text{is uncountable.}$$

Problem 2. Suppose that P and Q are true statements and R and S are false statements. Which of the following are given a truth value of TRUE?

1. $R \Longrightarrow P$
2. $(P \vee R) \wedge S$
3. $Q \Longrightarrow (P \Longrightarrow \neg S)$
4. $\neg(R \vee Q) \Longleftrightarrow S$

Problem 3. For a function $f : A \to B$, dene for $S \subset A$

$$f(S) = \{y \in B : y = f(x) \text{ for some } x \in S\}$$

Let $S, T \subset A$.

1. Prove that
$$f(S \cap T) \subset f(S) \cap f(T).$$
Give an example where these sets are not equal.

2. Prove that if f is an injection, then

$$f(S \cap T) = f(S) \cap f(T).$$

Problem 4. Prove that for all $n \in \mathbb{N}$

$$1 + 3 + 5 + 7 + \cdots + (2n - 1) = n^2.$$

Problem 5. For all of the following give a function $h : \mathbb{Z} \to \mathbb{Z}$ that satisfy the given properties:

1. h is injective but not surjective.
2. h is surjective but not injective.
3. h is a bijection.

Problem 6. Prove that if a and b are odd integers that $a^2 - b^2$ is divisible by 8.