Math 582

Homework - Part 1

Due March 17

Problem 1. Let μ be a Radon measure on $\mathbb{R}^n$. Prove that $\mu \ll \mathcal{H}^s$ if and only if $\Theta^{*,s}(\mu, x) < \infty$ for μ-a.e $x \in \mathbb{R}^n$.

Problem 2. For Radon measures μ and ν on $\mathbb{R}^n$ set

$$d(\mu, \nu) = \sum_{i=1}^{\infty} 2^{-i} \min\{1, F_i(\mu, \nu)\}.$$

Show that the space of Radon measures on $\mathbb{R}^n$ with the metric d defined above is a complete, separable metric space. Prove that the convergence with respect to d agrees with the weak convergence of Radon measures.