Math 582

Homework - Part 3

Due March 17

Problem 4. Let μ be a Radon measure on $\mathbb{R}^n$. Assume that for $a \in \text{spt } \mu = \Sigma$

$$1 \leq \limsup \frac{\mu(B(a, 2r))}{\mu(B(a, r))} < \infty. \tag{1}$$

1. Show that for $\tau \geq 1$ and $a \in \Sigma$

$$1 \leq \limsup \frac{\mu(B(a, \tau r))}{\mu(B(a, r))} < \infty.$$

2. Prove that if there exit $\kappa > 1$ and $R > 0$ such that for $r \in (0, R)$ and all $a \in \Sigma$

$$\frac{\mu(B(a, 2r))}{\mu(B(a, r))} \leq \kappa \tag{2}$$

then for all $\nu \in Tan(\mu, a)$ such that

$$\nu = \lim_{i \to \infty} (\mu(B(a, r_i)))^{-1} T_{a, r_i \#} \mu$$

$x \in \text{spt } \nu$ if and only if there exists a sequence $x_i \in T_{a, r_i}(\Sigma)$ such that $x_i \to x$.