Math 583

GEOMETRIC MEASURE THEORY IN EUCLIDEAN SPACE

AND TOPICS IN ERGODIC THEORY

Spring 2008

Instructors:

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TOPICS PART I:

1. Rectifiability and tangent measures (Preiss theorem)
2. Applications to harmonic analysis: In the plane the harmonic measure of a domain and that of its complement completely determine the size and the structure of the boundary. The Shannon-McMillan-Breiman Theorem has been the basis for several very interesting constructions which show that in higher dimensions the situation is more complicated. The discussion of these examples will lead to some of the open questions in the field. We will discuss several results as illustrations of the techniques developped during the first quarter.

RECOMMENDED LITERATURE PART I:

1. C. De Lellis, *Lecture notes on rectifiable sets, densities, and tangent measures*, Preprint 23, Institut für Mathematik, Universität Zurich, 2006.

2. C. Kenig, D. Preiss and T. Toro, Boundary structure and size in terms of interior and exterior harmonic measures in higher dimensions, to appear in JAMS.

3. O. Kowalski and D. Preiss, Besicovitch-type properties of measures and submanifolds, *J. Reine Angew. Math.* **379** (1987), 115–151.

4. J. Lewis, G. C. Verchota, & A. Vogel, On Wolff Snowflakes, *Pacific J. of Math*, **218** (2005), 139–166.

5. P. Mattila, *Geometry of Sets and Measures in Euclidean Spaces*, Cambridge University Press, 1995.

6. D. Preiss, Geometry of measures in $\mathbb{R}^n$: distribution, rectifiability, and densities, *Ann. of Math.* **125** (1987), 537–643.

7. T. Wolff, Counterexamples with harmonic gradients in $\mathbb{R}^3$, *Essays in honor of Elias M. Stein,* Princeton Mathematical Series **42** (1995), 321-384.

TOPICS PART II:

1. The fundamentals of Ergodic Theory: examples of measure-preserving transformations, ergodicity, recurrence, mixing.

2. Ergodic Theorems.

3. Entropy Theory, including the Shannon-McMillan-Breiman Theorem.

There are many links between Ergodic Theory and Geometric Measure Theory; in particular, "the dimension of an invariant measure and its entropy are equivalent quantities" (quoting from the paper by T. Wolff on harmonic measure in 3-space). However, this is a vast amount of material, and we will have to be very selective in our coverage.

RECOMMENDED LITERATURE PART II:

1. William Parry, *Topics in ergodic theory*, Cambridge University Press, 1981.

2. Karl Petersen, *Ergodic Theory*, Cambridge University Press, 1989.

3. Peter Walters, *An Introduction to Ergodic Theory*, Springer, 1981.

Homework due dates:

- Monday, May 5 (Toro)
- Monday, June 9 (Solomyak)