Math 524

Homework due 10/07/09

Reading from Stein & Shakarchi: Introduction, Chapter 1, §1,2,3.

Problem 1.

1. Let $\{x_n\}_n \subset \mathbb{R}$ be a bounded sequence. Show that $\limsup_{n\to\infty} x_n$ and $\liminf_{n\to\infty} x_n$ are the largest and smallest cluster points of the sequence $\{x_n\}$.

2. Show that every bounded sequence has a subsequence that converges to a real number.

Problem 2 Let $\{x_n\}_n \subset \mathbb{R}$, and $x \in \mathbb{R}$. Show that $x = \lim_{n\to\infty} x_n$ if and only if every subsequence of $\{x_n\}$ has in turn a subsequence which converges to x.

Exercises from Stein & Shakarchi: 1, 2 Chapter 1 (pages 37 and 38).