Math 524

Homework due 11/04/09

Reading from Stein & Shakarchi: Chapter 1, §4. Chapter 2, §1.

Exercises from Stein & Shakarchi: Chapter 1: exercises 35, 36, 37.
Chapter 6: exercise 1 (Please note that you also need to assume that $\mathcal{M}$ is closed under finite intersections.)

In problem 35 use the results of problem 34. It is not necessary to do problem 34.

Problem 1: Let $(X, \mathcal{M}, \mu)$ be a measure space. Assume that $f \in \mathcal{L}^+$ (i.e $f : X \to [0, \infty]$ measurable), and define
$$\lambda(E) = \int_E f \, d\mu \quad \text{for} \quad E \in \mathcal{M}.$$
Prove that λ is a measure on $\mathcal{M}$, and for $g \in \mathcal{L}^+$
$$\int g \, d\lambda = \int g f \, d\mu.$$

Hint: First suppose that g is simple.