Math 524

Homework due 12/02/09

Reading from Stein & Shakarchi: Chapter 6, §1, §2, and §3.

Exercises from Stein & Shakarchi: Chapter 6: 2, 3

Problem 1: If $f \geq 0$ be a Lebesgue integrable function on $[0,1]$. If for every $n = 1, 2, \cdots$

$$\int_0^1 [f(x)]^n \, dx = \int_0^1 f(x) \, dx,$$

show that there exists a Lebesgue measurable set $A \in [0,1]$ such that $f = \chi_A$ m-a.e..

Problem 2: Suppose $(X, \mathcal{M}, \mu)$ is a measure space, and $\mathcal{F}$ is a family of non-negative integrable functions on X with the following properties:

1. If $\varphi,\ \psi \in \mathcal{F}$, then $\varphi + \psi \in \mathcal{F}$.
2. If $\varphi,\ \psi \in \mathcal{F}$, then $\max\{\varphi, \psi\} \in \mathcal{F}$.
3. If f is measurable, $f \geq 0$ and $\int f \, d\mu > 0$, then there is $\varphi \in \mathcal{F}$ such that $\varphi \leq f$ and $\int \varphi \, d\mu > 0$.
4. $0 \in \mathcal{F}$.

Prove that if f is non-negative and measurable then

$$\int f \, d\mu = \sup\{\int \varphi \, d\mu : \varphi \in \mathcal{F}, \varphi \leq f\}$$