

## Syllabus Math 524-5-6

### Math 524

1. Introduction
  - (a) Real number system
  - (b) (Brief) review of metric spaces (complete and compact metric spaces)
2. Lebesgue Measure
  - (a) Rectangles and cubes in Euclidean space
  - (b) Exterior measure
  - (c) Measurable sets & Lebesgue measure (construction of a non-measurable set).
3. Abstract Measure Theory
  - (a) Exterior measure & Caratheodory's theorem
  - (b) Metric exterior measures
  - (c) Extension theorem
4. Integration Theory
  - (a) Measurable functions (approximation by step functions)
  - (b) Littlewood's 3 principles (Egoroff & Lusin)
  - (c) Integration of non-negative functions (Monotone convergence theorem, Fatou's theorem)
  - (d) Integration of extended real-valued functions (Lebesgue dominated convergence theorem)
  - (e) Product measure (Tonelli & Fubini)

## Math 525

### 1. Differentiation & Integration

- (a) Differentiation of an integral (Vitali covering lemma, Hardy-Littlewood maximal function)
- (b) Functions of bounded variation
- (c) Absolutely continuous functions (fundamental theorem of calculus)
- (d) Absolute continuity of functions

### 2. Hilbert Spaces

- (a) Definitions
- (b) Orthogonality
- (c) Unitary mappings
- (d) Closed subspaces & orthogonal projections
- (e) Linear transformations (Riesz Representation Theorem & Adjoints)
- (f) Application: Radon-Nikodym Theorem

### 3. Topology

- (a) Topological spaces (Hausdorff spaces)
- (b) Continuous maps (Urysohn and Tietze theorems)
- (c) Compact spaces
- (d) Compactness results for continuous function (Arzela- Ascoli)
- (e) Stone-Weierstrass Theorem

### 4. $L^p$ spaces and Banach spaces

- (a)  $L^p$  spaces
- (b) Banach spaces
- (c) The dual of  $L^p$
- (d) Hahn-Banach Theorem
- (e) Baire Category Theorem & Applications: Open Mapping Theorem, Closed Graph Theorem, Uniform Boundedness Principle

## Math 526

1. Weak convergence & Weak \* convergence : applications to  $L^p$  spaces.
2. The dual of  $C(X)$ 
  - (a) Positive linear functionals (Radon measures)
  - (b) Riesz Representation Theorem
3. Compact Operators
  - (a) Hilbert-Schmidt operators
  - (b) Fredholm alternative
  - (c) Spectrum of compact operators
  - (d) Symmetric operators
4. Elements of Fourier Analysis
  - (a) Preliminaries (Schwartz space)
  - (b) Convolutions
  - (c) The Fourier Transform
  - (d) Applications to PDE
5. Elements of Distribution Theory
  - (a) Distributions & Operations on distributions
  - (b) Supports of distributions
  - (c) Tempered distributions
  - (d) Fourier transform
  - (e) Sobolev spaces
  - (f) Applications to PDE