Math 525

Homework due 01/28/2015

Reading from Folland: Chapter 4: sections 2, 4, 5 and 6.

Problem 1: Let (X, d) be a metric space. Prove that if A and B are closed disjoint subsets of X there exist open disjoint sets U and V such that $A \subset U$ and $B \subset V$.

Problem 2: Let X be a topological space. Let $U \subset X$ be an open set. Prove that if A is dense in X then $\overline{U} = \overline{U \cap A}$.

Problem 3: Let X and Y be topological spaces. Prove that the following are equivalent:

1. $f : X \to Y$ is continuous.
2. $f(\overline{A}) \subset \overline{f(A)}$ for all $A \subset X$.
3. $\overline{f^{-1}(B)} \subset f^{-1}(\overline{B})$ for all $B \subset Y$.

Problem 4: Let X and Y be topological spaces. Assume that Y is Hausdorff, and $f, g : X \to Y$ are continuous. Prove

1. $\{x \in X : f(x) = g(x)\}$ is closed in X.
2. If $f = g$ on a dense subset of X then $f = g$ on all of X.

Problem 5:

1. Let X and Y be topological spaces, let X be compact and $\pi_2 : X \times Y \to Y$ denote the projection onto Y. Let $A \subset X \times Y$ be a closed subset. Show that $\pi_2(A)$ is a closed subset of Y.
2. Let X and Y be topological spaces, and assume that Y is Hausdorff. Prove that if $f : X \to Y$ is continuous then $\mathrm{graph\,f}$ is a closed subset of $X \times Y$. Here

$$\operatorname{graph} f = \{(x, f(x)) : x \in X\}.$$

Show by an example that the converse is false.