Math 525

Homework due 02/18/2015

Reading from Folland, 2nd edition: Chapter 5: sections 3 and 5.

Problems from Folland: Chapter 5: problems 17,19, 22, 25.

Problem 1: Suppose that the Banach space $\mathcal{X}$ is uniformly convex. That is, suppose that for every $\epsilon > 0$ there exists $\delta \in (0,1)$ so that if

$$\|x\| = \|y\| = 1 \text{ and } \|x - y\| \geq \epsilon \text{ then } \|\frac{x+y}{2}\| < 1 - \delta.$$

Let f be a linear functional on $\mathcal{X}$ with norm 1. Prove that there is a unique point $x \in \mathcal{X}$ such that $\|x\| = 1$ and $f(x) = 1$.