GEOMETRIC MEASURE THEORY

HOMEWORK

Problem 1 (Intersection with half spaces): If $H_t = \{x \in \mathbb{R}^n : \langle x, r \rangle < t\}$ for some $e \in \mathbb{S}^{n-1}$, $t \in \mathbb{R}$, and E a set of finite perimeter in $\mathbb{R}^n$, then $E \cap H_t$ is a set of finite perimeter in $\mathbb{R}^n$. Show that for a.e. $t \in \mathbb{R}$

$$\nu_{E \cap H_t} \|\partial(E \cap H_t)\| = \nu_E \|\partial E\| \llcorner H_t + e\mathcal{H}^{n-1} \llcorner (E \cap \partial H_t).$$

Note that for a.e. $t \in \mathbb{R}$

$$\mathcal{H}^{n-1}(E \cap \partial H_t) \leq \|\partial E\|(H_t), \qquad \|\partial(E \cap H_t)\|(\mathbb{R}^n) \leq \|\partial E\|(\mathbb{R}^n).$$

Problem 2 (Convex sets are of locally finite perimeter):

1. Show that a set is convex if and only if its closure is the intersection of countably many closed half spaces.
2. Show that, if E is a set of locally finite perimeter in $\mathbb{R}^n$, and if K is convex, then $E \cap K$ is a set of locally finite perimeter, with $\|\partial(E \cap K)\|(\mathbb{R}^n) \leq \|\partial E\|(\mathbb{R}^n)$.
3. Show that every convex set is of locally finite perimeter, and every bounded convex set is of finite perimeter.