PROBLEMS WEEK 1

Problem 1: Prove that if μ is a Borel regular measure on $\mathbb{R}^n$, A is a μ-measurable set and $\mu(A) < \infty$ then $\mu \llcorner A$ is a Radon measure.

Problem 2: Let μ be a Borel measure in $\mathbb{R}^n$ and B be a Borel set. Prove:

1. If $\mu(B) < \infty$, $\forall \epsilon > 0$ there exists a closed subset $C \subset \mathbb{R}^n$ such that $C \subset B$ and $\mu(B \backslash C) < \epsilon$.
2. If μ is a Radon measure, $\forall \epsilon > 0$ there exists an open set $U \subset \mathbb{R}^n$ such that $B \subset U$ and $\mu(U \backslash B) < \epsilon$.

Problem 3: Prove the approximation by open and compact sets for Radon measures. Let μ be a Radon measure in $\mathbb{R}^n$, then

1. For each set $A \subset \mathbb{R}^n$
$$\mu(A) = \inf\{\mu(U) : A \subset U,\ U \text{ open}\}$$
2. For each μ-measurable set $A \subset \mathbb{R}^n$
$$\mu(A) = \sup\{\mu(K) : K \subset A,\ K \text{ compact}\}$$

Problem 4: Prove that if μ is a Borel regular measure in $\mathbb{R}^n$ its support

$$\operatorname{spt} \mu = \{x \in \mathbb{R}^n : \mu(B(x,r)) > 0,\ \forall r > 0\}$$

is a closed set.