Problem 1: Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a Lipschitz map, and $A \subset \mathbb{R}^n$ be an $\mathcal{H}^n$-measurable set. Show that $\Theta^n_*(f(A), x) > 0$ for $\mathcal{H}^n$ almost every $x \in f(A)$.

Definition 1: A map $f : A \to B$, $A \subset \mathbb{R}^n$, $B \subset \mathbb{R}^m$ is said to be bi-Lipschitz if f is Lipschitz and it has a Lipschitz inverse $f^{-1} : B \to A$.
Definition 2: A set $E \subset \mathbb{R}^n$ is said to be an Ahlfors s-regular set for some $0 < s \leq n$, if there exists a constant $C > 1$ so that for every $r > 0$ and each $x \in E$,

$$C^{-1} r^s \leq \mathcal{H}^s(E \cap B(x, r)) \leq C r^s.$$

Problem 2: Show that the image of an Ahlfors s-regular set by a bi-Lipschitz map is an Ahlfors s-regular set.

Problem 3: Let $S \subset \mathbb{R}^n$, $m \leq n - 1$, and $\epsilon \in (0, \frac{1}{2})$. Let $0 \in S$. Assume that there exists an m plane L containing the origin, such that $\forall \rho \in (0, 1]$ and for each $x \in S \cap B(0, 1)$

$$S \cap B(x, \rho) \subset (\epsilon\rho) - \text{neighborhood of } (L + x) \cap B(x, \rho).$$

Prove that $S \cap B(0, \frac{1}{4})$ is contained in a Lipschitz graph. Give an estimate for the Lipschitz constant of the corresponding function.

Problem 4: Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be Lipschitz, $n \geq m$. Let $g : \mathbb{R}^n \to \mathbb{R}$ be an $\mathcal{H}^n$-summable function. Assume that $\sup_{x \in \mathbb{R}^n} |f(x)| \leq R$, and that $g \geq 0$. Show that for each $\mathcal{H}^n$-measurable set $A \subset \mathbb{R}^n$, there exists a set $S \subset B(0, R) \subset \mathbb{R}^m$ ($S = S(g, f, A)$), such that $\mathcal{H}^m(S) \geq \frac{1}{2}\mathcal{H}^m(B(0, R))$, and for each $y \in S$

$$\int_{f^{-1}(y) \cap A} g \, d\mathcal{H}^{n-m} \leq \frac{2}{\mathcal{H}^m(B(0, R))} \int_A g \, Jf \, d\mathcal{H}^n.$$

Problem 5: If E, $F \subset \mathbb{R}^n$ are sets of (locally) finite perimeter, then $E \cap F$ and $E \cup F$ are sets of (locally) finite perimeter, and, for $U \subset \mathbb{R}^n$ open

$$\|\partial(E \cup F)\|(U) + \|\partial(E \cap F)\|(U) \leq \|\partial E\|(U) + \|\partial F\|(U).$$