TOPICS IN GEOMETRIC MEASURE THEORY

Week 3

- Read Chapter 15 in Maggi.
- **Problem 1 (Intersection with half spaces):** If $H_t = \{x \in \mathbb{R}^n : \langle x, e\rangle < t\}$ for some $e \in \mathbb{S}^{n-1}$, $t \in \mathbb{R}$, and E a set of finite perimeter in $\mathbb{R}^n$, then $E \cap H_t$ is a set of finite perimeter in $\mathbb{R}^n$. Show that for a.e. $t \in \mathbb{R}$

$$\mu_{E\cap H_t} = \mu_E \llcorner H_t + e\mathcal{H}^{n-1} \llcorner (E \cap \partial H_t).$$

  Note that for a.e. $t \in \mathbb{R}$

$$\mathcal{H}^{n-1}(E \cap \partial H_t) \leq \mu_E(H_t), \qquad |\mu_{E\cap H_t)}|(\mathbb{R}^n) \leq |\mu_E|(\mathbb{R}^n).$$

- **Problem 2:** If $E \subset \mathbb{R}^n$ is a set of locally finite perimeter and $x \in \partial^* E$ then

$$\nu_E(x) = \lim_{r\to 0^+} \frac{1}{\omega_{n-1} r^{n-1}} \int_{B(x,r)\cap\partial^* E} \nu_E \, d\mathcal{H}^{n-1}.$$

- Read the introduction to Chapter 16 in Maggi.
- Read section 16.1 in Maggi.