## Math 135 - Winter 2000

### Homework to be done by January 6th:

**Section 10.1:** Problems 1, 3, 5, 7, 9, 11, 27, 28.
**Section 10.2:** Problems 1, 3, 7, 11, 13, 17, 18, 21, 47, 55, 57, 59, 61.
**Section 10.3:** Problems 6, 9, 12, 17, 23, 31, 33, 47.

### Problems to be handed in on January 6th

**1.** Prove that if $a$ and $b$ are any two positive numbers with $a < b$, then there exists a rational number $r$ such that $a < r < b$. (Hint: Show that there is a positive integer $n$ such that $na > 1$ and $n(b-a) > 1$. The show that there is a positive integer $m$ such that $m - 1 \leq na < m$. Let $r = \frac{m}{n}$.

**2.** Let $\{a_n\}$ and $\{b_n\}$ be sequences such that $a_n \to 0$ and $\{b_n\}$ is bounded. Prove that $a_n b_n \to 0$.

**3.** Suppose that $f$ is a differentiable function on $(0, \infty)$ such that $f'(x) \to 0$ as $x \to \infty$. Show that

$$\lim_{n\to\infty} [f(n+1) - f(n)] = 0$$

For example $\sqrt{n+1} - \sqrt{n} \to 0$ as $n \to 0$ even though $\sqrt{n} \to \infty$ as $n \to \infty$. Hint: You have already done this problem.