Math 135 - Winter 2000

Homework to be done by January 27th

Section 11.4: Problems 2, 3, 5, 7, 11, 12, 14, 15, 17, 19, 33, 42, 45.
Section 11.5: Problems 1, 3, 5, 6, 7, 9, 11, 13 ,16, 27, 31, 33, 36, 42, 50, 53.
Section 11.6: Problems 1, 3, 7, 11, 13, 17, 18, 23, 33.

Problems to be handed in on January 27th

1.1 Show that

$$e^{-x^2} = 1 - x^2 + \frac{1}{2}x^4 + R(x),$$

where

$$|R(x)| \le \frac{1}{6}x^6 \quad \text{for all} \quad x \in \mathbf{R}.$$

1.2 Use 1.1 to show that

$$\int_0^{1/2} e^{-x^2}\,dx = 0.461,$$

with an error less than 0.001.

2.1 Show that

$$|\ln(1+x) - x| \le 2x^2 \quad \text{for} \quad |x| \le \frac{1}{2}.$$

2.2 Suppose that $a_k \ge 0$ for all $k \ge 1$. Show that

$$\sum_{k=1}^{\infty} \ln(1 + a_k)$$

converges if and only if

$$\sum_{k}^{\infty} a_k$$

converges.

1.3 Does the series

$$\sum_{k=1}^{\infty} (-1)^k \ln\left(\frac{(k+1)^2}{k^2+1}\right)$$

converges absolutely'? Conditionally? Not at all? Justify your answers.

Optional Construct an example of a sequence $\{a_k\}$ with alternating sign such that $\sum_{k=1}^{\infty} a_k$ converges but $\sum_{k=1}^{\infty} \ln(1 + a_k)$ diverges. (Thus the hypothesis $a_k \ge 0$ in 1.2 cannot be omitted.)

The first midterm will be on Friday January 28th. It will cover Chapter 10 and Sections 11.1-4.