Math 135 - Winter 2000

Homework to be done by February 3rd

Section 11.7: Problems 3, 4, 5, 6, 7, 10, 12, 20, 25, 30, 31, 34, 46.
Section 11.8: Problems 1, 3, 4, 5, 8, 9, 10, 13, 15, 19, 27, 28, 41, 44.

Also, the following problems related to the material in the notes "More on Taylor Polynomials." Answers are on the back of this page.
In Problems 1–6, find the 5th order Taylor polynomial (about $a = 0$) of the given function by using the known expansions for e^x, $\sin x$, $\cos x$, and $(1-x)^{-1}$, together with the techniques discussed in the notes.

1. $(x^2+1)e^x$
2. $\frac{\cos x}{1-x}$
3. $\cos(x^3)$
4. $\frac{1+x}{1+x^2}$
5. $\sin(x+x^2)$
6. $\cos(e^x-1)$

In problems 7–9, compute the indicated limits by using Taylor expansions of the numerator and denominator, as discussed in the notes.

7. $\displaystyle\lim_{x\to 0} \frac{e^x - e^{-x}}{\sin 5x}$ 8. $\displaystyle\lim_{x\to 0} \frac{1+2x+2x^2-e^{2x}}{\sin x - x}$ 9. $\displaystyle\lim_{x\to 0} \frac{\sin^4 x}{1-\cos(x^2)}$

Problems to be handed in on February 3rd

Problem 1.
1.1 Find the 4th order Taylor polynomial (about $a = 0$) of $\cos(\sin x)$.

1.2 Use the result of (1.1) (not l'Hospital's rule) to compute

$$\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^2 \sin(x^2)}.$$

Problem 2.
2.1 Find all values of x for which the following series converges (i) absolutely, (ii) conditionally:

$$f(x) = \sum_{0}^{\infty} \frac{(-1)^n (x-3)^n}{n+2}.$$

2.2 Show that the function f defined in (2.1) satisfies the differential equation

$$(x-3)f'(x) + 2f(x) = \frac{1}{x-2}$$

on its interval of convergence.

2.3 Either working directly from (2.1) or using (2.2), express $f(x)$ in closed form (i.e., no infinite series — it's an elementary function).

ANSWERS TO PROBLEMS ON THE OTHER SIDE OF THE PAGE

1. $1 + x + \frac{3}{2}x^2 + \frac{7}{6}x^3 + \frac{13}{24}x^4 + \frac{7}{40}x^5$
2. $1 + x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{13}{24}x^4 + \frac{13}{24}x^5$
3. 1
4. $1 + x - x^2 - x^3 + x^4 + x^5$
5. $x + x^2 - \frac{1}{6}x^3 - \frac{1}{2}x^4 - \frac{59}{120}x^5$
6. $1 - \frac{1}{2}x^2 - \frac{1}{2}x^3 - \frac{1}{4}x^4 - \frac{1}{24}x^5$
7. $\frac{2}{5}$
8. 8
9. 2