Math 135 - Winter 2000

Homework to be done by February 10th

Section 18.3: 1, 3,, 4, 5, 9, 21, 23, 34, 35.

The following problems are related to the material in the various notes that will be handed out this week. Answers are on the back of this page.

1. Derive the addition formulas for sine and cosine from the formulas

$$\cos x = (e^{ix} + e^{-ix})/2 \quad \text{and} \quad \sin x = (e^{ix} - e^{-ix})/2i$$

as sketched in the notes.

2. Show that $\sum_0^\infty \frac{\sin k}{2^k}$ converges. Evaluate it exactly by using the fact that $\sin k = \text{Im}(e^{ik})$.
Hint: it will be useful to recall that

$$\frac{1}{a+ib} = \frac{a-ib}{a^2+b^2} = \frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2} \qquad (a, b \text{ real}). \qquad (*)$$

3. Let y_1, y_2 be solutions of $y'' + py' + qy = 0$ where p, q are continuous functions on the interval I. Show that there cannot be a point in I where y_1 and y_2 both vanish, or where they both have maxima or minima, unless one of them is a constant multiple of the other.

4. Show that $y_1 = x^2$ and $y_2 = x^4$ are both solutions of $y'' - 5x^{-1}y' + 8x^{-2}y = 0$. They both vanish and both have minima at $x = 0$.

In Problems 5–7, you are given a differential equation and one solution of it. Use the "reduction of order" trick to find the general solution.

5. $x^2y'' - x(x+2)y' + (x+2)y = 0$, $y_1 = x$.

6. $xy'' - (x+2)y' + 2y = 0$, $y_1 = e^x$.

7. $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0$, $y_1 = x^{-1/2} \sin x$.

Problems to be handed in on February 10th

Problem 1

1.1 Verify that the formula $d(e^{cx})/dx = ce^{cx}$ remains valid when $c = a + ib$ is a complex number.

1.2 It follows that the integration formula $\int e^{cx}dx = c^{-1}e^{cx} + C$ is also valid for complex c. Use this to give a quick derivation (no integration by parts!) of the formulas for $\int e^{ax}\cos bx\,dx$ and $\int e^{ax}\sin bx\,dx$. (Cf. formula $(*)$ on the other side of the page.)

Problem 2 Suppose y_1 and y_2 are a fundamental set of solutions of $y'' + p(x)y' + q(x)y = 0$ on the interval I, where p and q are continuous functions on I. Let $z_1 = ay_1 + by_2$ and $z_2 = cy_1 + dy_2$ where a, b, c, d are constants. Show that z_1 and z_2 are a fundamental set of solutions if and only if $ad - bc \neq 0$.

Answers to problems on the other side

2. $\frac{2\sin 1}{5+4\cos 1}$
5. $x(C_1 + C_2e^x)$
6. $C_1e^x + C_2(x^2 + 2x + 2)$
7. $x^{-1/2}(C_1\sin x + C_2\cos x)$