Math 135 - Winter 2000

Homework to be done by February 17th

Section 18.4: Problems 3, 5, 6, 8, 9, 23, 26, 29, 31, 35.
Section 18.5: 13, 14, 15.

The following problems relate to the material in the notes. Answers are on the back of the page.

In Problems 1–3, find the general solution by variation of parameters. (The associated homogeneous equations were solved in last week's assignment. Remember that equations have to be put in standard form for the formulas in variation of parameters to work.)

1. $x^2y'' - x(x+2)y' + (x+2)y = 2x^3$.
2. $xy'' - (x+2)y' + 2y = x^3e^x$.
3. $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 4x^{3/2}\sin x$.

Problems 4–6 deal with Euler equations on the interval $0 < x < \infty$.

4. Find the general solution of $x^2y'' + 8xy' + 12y = 0$.
5. Find the general solution of $x^2y'' - 5xy' + 9y = 0$.
6. Solve the initial value problem $x^2y'' + 3xy' + 5y = 0$, $y(1) = 2$, $y'(1) = -2$.

In Problems 7–10, either compute the radius of convergence of the power series solutions about $x_0 = 0$ or give a lower bound for it. In Problems 7 and 8, also compute the terms of these series up through x^5.

7. $e^xy'' + xy = 0$.
8. $(4-x)y'' + 8y = 0$.
9. $y'' + [\log(1-x)]y' + (3+x)^{-1}y = 0$.
10. $(x^2 + 4x + 7)y'' + xy' - y = 0$.

(There will be more problems on series solutions next week.)

THE HAND-IN PROBLEMS ARE ON THE BACK SIDE.

Problems to be handed in on February 17th

Problem 1. Consider a damped mass-spring system whose natural motion is governed by the equation $y'' + 2y' + 5y = 0$, where y is the displacement from equilibrium and the independent variable is the time t. The system is subjected to a sinusoidal force of frequency ω and unit amplitude, so the equation of motion is

$$y'' + 2y' + 5y = \sin \omega t.$$

1.1 Find the solution of this equation with $y(0) = y'(0) = 0$. Express it in the form $A\sin(\omega t+\phi)+\widetilde{y}$ where A and ϕ are suitable constants and $\widetilde{y}$ decays exponentially as $t \to \infty$. (This describes the situation for $t > 0$ when the system starts up from rest at time $t = 0$; the motion is a sine wave of frequency ω and amplitude A plus a transient response to the initial conditions that dies out for large t.)

1.2 Find the value of ω that maximizes the amplitude A. (This value is called the *resonant frequency* of the system.)

Problem 2. Solve the initial value problem

$$x^2y'' + 5xy' + 3y = \pi^2 \cos \pi x, \quad y(1) = 0, \quad y'(1) = 1,$$

on the interval $0 < x < \infty$.

ANSWERS TO PROBLEMS ON THE OTHER SIDE

1. $y = c_1 x + c_2 x e^x - 2x^2$.
2. $y = c_1 e^x + c_2(x^2 + 2x + 2) + \frac{1}{3}x^3 e^x$.
3. $y = x^{-1/2}(c_1 \sin x + c_2 \cos x) - 2x^{1/2} \cos x$.
4. $y = c_1 x^{-3} + c_2 x^{-4}$.
5. $y = c_1 x^3 + c_2 x^3 \log x$.
6. $y = 2x^{-1} \cos(\log x)$.
7. $r = \infty$; $y = a_0[1 - \frac{1}{6}x^2 + \frac{1}{12}x^4 - \frac{1}{40}x^5 + \cdots] + a_1[x - \frac{1}{12}x^4 + \frac{1}{20}x^5 + \cdots]$.
8. $r \geq 4$; $y = a_0[1 - x^2 - \frac{1}{12}x^3 + \frac{5}{32}x^4 + \frac{61}{1920}x^5 + \cdots] + a_1[x - \frac{1}{3}x^3 - \frac{1}{24}x^4 + \frac{13}{480}x^5 + \cdots]$.
9. $r \geq 1$.
10. $r \geq \sqrt{7}$.