1 (10 pts).

(a) (3 pts) $\lim_{x\to\infty} \ln\left(\frac{x}{x+1}\right) = \lim_{x\to\infty} \ln\left(\frac{1}{1+\frac{1}{x}}\right) = \ln(1) = 0$

(b) (2 pts) $\lim_{x\to-\infty} \frac{2x^4 - x}{x^3+1} = \lim_{x\to-\infty} x\,\frac{2-\frac{1}{x^3}}{1+\frac{1}{x^3}} = -\infty$

(c) (3 pts) $\lim_{x\to 2} \frac{\sqrt{x+2}-2}{x-2}\cdot\frac{\sqrt{x+2}+2}{\sqrt{x+2}+2} = \lim_{x\to 2}\frac{x-2}{(x-2)(\sqrt{x+2}+2)} = \lim_{x\to 2}\frac{1}{\sqrt{x+2}+2} = \frac{1}{4}$

(d) (2 pts) ∞

2 (9 pts).

(a) (4 pts) $P = \frac{10}{\sqrt{2}}(\sqrt{2})^t = 10\cdot 2^{\frac{t-1}{2}}$.

(b) (5 pts) $P = 50\sin\left(\frac{\pi}{6}(t-3)\right) + 70$.

3 (10 pts). (a) $f(2) = 0$; (b) limit not exists since $\lim_{x\to 4^-} f(x) = 3 \neq \lim_{x\to 4^+} f(x) = 4$; (c) $-\infty$; (d) $\frac{f(4)-f(2)}{4-2} = 3/2$; (e) $-3/2$

4 (9 pts).

(a) OB is the hypotenuse of a right-triangle with length 2 and opposite side length 1, so OB makes angle $\pi/6$ with the horizontal. Since Hobbs started at A, then in 5 secs he rotates an angle of $\pi/2 + \pi/6 = 2\pi/3$ radians. Thus angular velocity (in counterclockwise direction) is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi/3}{5} = \frac{2\pi}{15}.$$

$\theta_0 = \frac{3\pi}{2}$ (or $-\frac{\pi}{2}$). Thus, $\theta(t) = \frac{3\pi}{2} + \frac{2\pi}{15}t$. The circle is centered at $(0,0)$ with radius $r = 2$, so

$$x(t) = 0 + 2\cos\left(\frac{3\pi}{2} + \frac{2\pi}{15}t\right), \qquad y(t) = 0 + 2\sin\left(\frac{3\pi}{2} + \frac{2\pi}{15}t\right).$$

(b) (3 pts) Hobbs moves a distance of 1 meter along the perimeter of the circle. Since the circumference is $C = 2\pi r = 4\pi$ meters, this means Hobbs moves along $\frac{1}{4\pi}$th of the circle, corresponding to an angle of $2\pi\cdot\frac{1}{4\pi} = 1/2$ radians. Thus $\Delta t = \frac{\Delta\theta}{\omega} = \frac{1/2}{2\pi/15} = \frac{15}{4\pi}$ secs.

5 (12 pts). $C(t) = t/(t+2)$.

(a) (2 pts) $\frac{C(a+h)-C(a)}{a} = \frac{\frac{a+h}{a+h+2} - \frac{a}{a+2}}{h}$.

(b) (6 pts) Simplifying (a) to $\frac{2}{(a+h+2)(a+2)}$ and taking the limit as $h\to 0$ yields $\frac{2}{(a+2)^2}$.

(c) $t/(3t+2)$.

(d) Solving $C = t/(t+2)$ for t yields $t = 2C/(1-C)$.