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MATHEMATICS, DIDACTICAL ENGINEERING AND OBSERVATION

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TEXT (SLIGHTLY ANNOTATED) OF POWERPOINT PRESENTATION OF PLENARY LECTURE

ENGLISH TRANSLATION : GINGER WARFIELD

1. MATHEMATICS, PROBLEMS AND SITUATIONS

- ***CLASSICAL PROBLEMS*** **ARE** ***GENERATED*** **BY ACTING IN VARIOUS WAYS ON THEOREMS OR CLOSED STATEMENTS.**

- **H IMPLIES C →**
- **- GIVEN H AND C, PROVE H ⇒ C**
- **- GIVEN H, LOOK FOR C**
- **- GIVEN C, LOOK FOR H**

ETC.

- **THEOREMS AND PROBLEMS ARE TWO FORMS OF A SAME MATHEMATICAL KNOWLEDGE**
- **THERE IS THUS NO MATHEMATICAL DIFFERENCE BETWEEN A THEOREM AND A PROBLEM: A THEOREM THAT HAS BEEN TAUGHT IS A PROBLEM THAT CAN SERVE AS A REFERENCE AND A PROBLEM IS A THEOREM THAT MUST BE RE-PROVED EVERY TIME IT IS ENCOUNTERED: THE DIFFERENCE IS THE DIDACTICAL ORDER.**
- **WHEN IT IS NECESSARY TO THINK ABOUT A THEOREM IN A SIGNIFICANT WAY, IN A PROBLEM OR IN A TEXT — IN OTHER WORDS WHEN IT IS NOT JUST BEING CITED — THE PROCESS OF PRODUCING IT IS A SAME KIND OF** ***MENTAL AND MATHEMATICAL ACTIVITY AS*** **FOR ITS FIRST INVENTOR.**

•EVEN IN A CALCULATION, THE USE OF A FORMULA LIKE:

$$2\cos X = (e^{ix} + e^{-ix})$$

REQUIRES MAKING SOME USE OF THE MEANING ITSELF — A LITTLE OF EULER'S THINKING.

•STRICTLY DEDUCTIVE THEORIES AND PROOFS DO NOT PROVIDE A GOOD DESCRIPTION OF THIS ACTUAL MATHEMATICAL ACTIVITY.

•AT BEST THEY PRESENT ONLY THE RESULT OF IT.

•SITUATION: A SYSTEM OF CONDITIONS THAT MAKE IT PROBABLE THAT A GROUP OF STUDENTS OR AN INSTITUTION WILL PRODUCE A THEOREM OR THE SOLUTION OF A PROBLEM.

•THE NOTION OF "SITUATION" INCLUDES, EXTENDS, ENLARGES AND DIVERSIFIES THE NOTION OF "PROBLEM":

•A PROBLEM IS A SITUATION WHOSE ONLY VISIBLE EXPRESSED DIDACTICAL VARIABLES ARE MATHEMATICAL CONDITIONS.

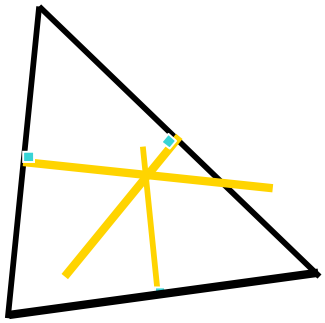
•I AM GOING TO GIVE AN EXAMPLE OF THE TRANSFORMATION OF A PROBLEM INTO A SITUATION THAT GIVES THE TEACHER A CHANCE TO BRING OUT QUESTIONS AND DISCUSSIONS FROM HER CLASS. YOU WILL SEE HOW THE PROCESS ALTERNATELY ENCOURAGES INDIVIDUAL AND COLLECTIVE EFFORT. BOTH ARE ESSENTIAL.

EXAMPLE : PROBLEM AND SITUATION

•CLASSICAL PROBLEM:

" PROVE THAT THE PERPENDICULAR BISECTORS OF THE SIDES OF ANY TRIANGLE ARE CONCURRENT"

•Usually, students draw or have available to them a figure in which the three perpendicular bisectors visibly intersect at a point.



The issue then is to prove something about which there is no doubt.

FIRST STUDENT : "I SEE IT! WHY GIVE A PROOF?"

The only students who feel the need of a proof are those who think it's a bit miraculous that the third perpendicular bisector manages to hit the point of intersection of the first two.

SECOND STUDENT: "IT'S WEIRD THAT THE THIRD PERPENDICULAR BISECTOR MANAGES TO HIT THE POINT OF INTERSECTION OF THE FIRST TWO"

The second called the text into question and spontaneously converted the situation from a simple problem into a situation with richer possibilities.

- **BUT TO MAKE THE MOST OF THIS ASTONISHMENT THEY NEED TO BE ABLE TO ENVISION ALTERNATIVE… AND THE QUESTION NEEDS TO CONCERN THE WHOLE CLASS.**

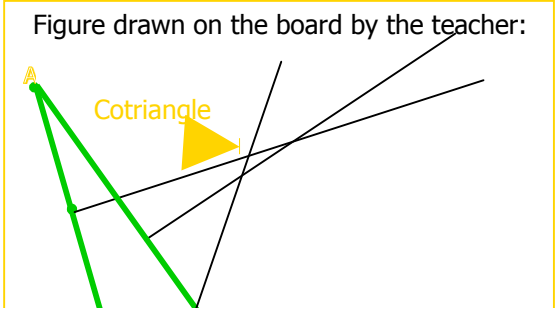
THUS THE TEACHER NEEDS TO IMAGINE, ORGANIZE AND PRODUCE A *MATHEMATICAL SITUATION* THAT WILL:

1. **ALLOW HER STUDENTS TO ENVISION ALTERNATIVES, AND**
2. **INVOLVE THE WHOLE CLASS**

HER OWN ROLE IN IT REQUIRES A *DIDACTICAL SITUATION*.

- A ***mathematical situation***, for the students:
- The teacher draws on the board a triangle and its three perpendicular bisectors.
- Drawing freehand (or with a slightly falsified T-square), she produces three points of intersection, and hence a small triangle!!!

Figure drawn on the board by the teacher:



To accredit the idea of the small triangle, she labels it "cotriangle" and requires the students to label its vertices.

For each student the teacher has prepared a sheet with an appropriate triangle (very obtuse, but with its center on the sheet)

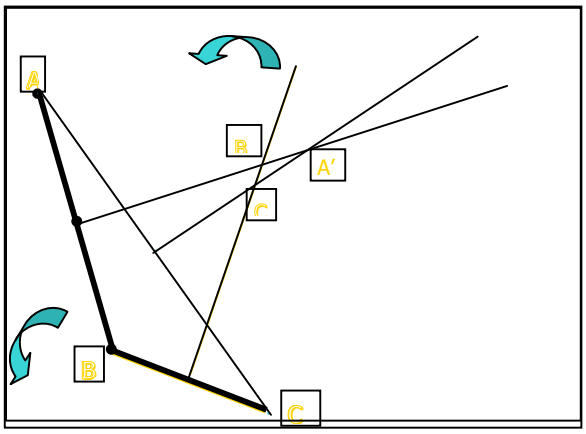
Assignment: Draw the perpendicular bisectors of the sides of this triangle

Result:
The students all get “cotriangles” that are small or even reduced to a point.

- The teacher looks surprised: “Hops…?
- All of you got little tiny cotriangles or even just points?
- She apologizes for having accidentally prepared a “special case” that is useless for studying cotriangles…
- and she assigns the students the task of producing a more satisfactory figure:

- **“DRAW A TRIANGLE SUCH THAT THE *THREE POINTS OF INTERSECTION OF THE PERPENDICULAR BISECTORS* ARE AS FAR AS POSSIBLE FROM EACH OTHER”.**

The students first think they can enlarge A’B’C’ by modifying the position of BC



- Repeated failures finally suggest the idea that the three points might just reresent one point.

- **MAYBE THE THREE POINTS “REPRESENT” ONLY ONE?**
- **HOW CAN WE BE SURE?**
- **FOR THAT TO BE KNOWN IT MUST BE “PROVED” INTELLECTUALLY, BECAUSE SOME OF THE DRAWINGS APPEAR TO INDICATE THE CONTRARY…**
- **WHICH LEADS TO A DEBATE THAT CALLS FORTH MATHEMATICAL ACTIVITY: DEFINITIONS, CONSEQUENCES, PROOFS, CHOICES OF AXIOMS,…**
- **BECAUSE THE STUDENTS HAVE TRY TO CONSTRUCT THEY NO LONGER TRYING TO WORK WITH THE WHOLE FIGURE THE TEACHER DREW, BUT INSTEAD ARE TRYING TO UNDERSTAND IT IN TERMS OF ITS CONSTRUCTION.**
- **BY FOLLOWING AN ORDER OF CONSTRUCTION RATHER THAN EXAMINING A COMPLETE, FIXED FIGURE, THE STUDENTS:**

- **FIRST CONSTRUCT THE PERPENDICULAR BISECTOR OF AB,**
- **THEN THAT OF AC, WHICH WILL DETERMINE A'.**
- **THEN WONDER WHETHER THAT OF BC CAN PASS ELSEWHERE THAN THROUGH A'.**

- **IN THE END, FOR THE STUDENTS,**

- **DEDUCTIVE GEOMETRY BECOMES A MEANS OF ESTABLISHING "WHAT HAS TO BE TRUE" AND NOT A DESCRIPTION OF WHAT CAN BE SEEN…**

- **THE SITUATION AS ENVISAGED PROVIDES THE TEACHER WITH A DIFFICULT ROLE TO PLAY:**

- **SHE MUST DRAW A FALSE FIGURE**

- **SHE MUST PRETEND TO HAVE MADE A MISTAKE AND EXCUSE HERSELF**

- **SHE MUST MAKE FALSE STATEMENTS**

- **SHE MUST GIVE FANTASY EXPLANATIONS FOR PARTICULAR CASES**

- **SHE MUST REQUIRE AND ENCOURAGE THE STUDENTS TO CARRY OUT IMPOSSIBLE TASKS**

- **SHE MUST CHANGE HER MIND IN AN UNEXPECTED WAY AND ADMIT IMPLICITLY TO HAVING LIED ON PURPOSE.**

- **SHE MUST LET HERSELF BE CAUGHT RED-HANDED CYNICALLY MANIPULATING HER STUDENTS.**

- **AFTER ALL THAT SHE MUST PROPOSE AN IMPROBABLE HYPOTHESIS: A THREE-IN-ONE POINT, AND GET THEM TO DISCUSS IT.**

- **FINALLY, SHE MUST EXPECT OF THE STUDENTS SOMETHING UNKNOWN TO THEM: TO PROVE SOMETHING — AND THEY ARE SUPPOSED TO DO IT IN A DIFFICULT CASE:**

PROOF BY CONTRADICTION.

- **AND SHE MIGHT EVEN HAVE**

AN UNCOOPERATIVE CLASS !

- **THE *ACTUAL REALIZATION* OF THE *SITUATION ENVISAGED* REQUIRES SOME CONDITIONS THAT AREN'T ALWAYS MET: A GOOD RELATIONSHIP BETWEEN**

THE TEACHER AND THE CLASS, A REAL ACTING TALENT ON THE TEACHER'S PART, AND A BIT OF LUCK…

- **OBVIOUSLY, ONE CANNOT REQUIRE EVERY TEACHER TO BE ABLE TO PLAY THIS DRAMATIC ROLE SUCCESSFULLY AND MAINTAIN SUCH A DELICATE DIDACTICAL CONTRACT WITH WHATEVER STUDENTS HAPPEN TO BE THERE.**

But it is not impossible to do – I have done it myself several times.

- **THE INTRODUCTION OF THE COTRIANGLE AND OF A FALSIFIED T-SQUARE MIGHT APPEAR SHOCKING.**

- **BUT THREE RADAR STATIONS REALLY DO PRODUCE A TRIANGLE OF UNCERTAINTY,**

- **AND NO PHYSICAL T-SQUARE IS PERFECT…**

THE RELATIONSHIP OF GEOMETRY WITH PRACTICAL SPACE IS THAT OF A MODEL TO ITS OBJECT. MATHEMATICAL GEOMETRY DESCRIBES THE CONSISTENCY OF OUR REASONING, *NOT* REALITY.

THE *NOTION OF SITUATION* ENABLES US :

A. **TO ORGANIZE THE CONDITIONS OF BEHAVIOR OF THE TEACHERS, THE STUDENTS AND THE MILIEU INTO A SYSTEM AND IN A MORE DETAILED AND COHERENT WAY THAN PROBLEMS,**
B. **TO CALCULATE THE VALUE OF THE DIFFERENT VARIABLES AND MORE EFFECTIVE CONDITIONS**
C. **TO MODEL THE EVOLUTION OF THIS SYSTEM BY CONSIDERING OBJECTIVELY THE BEHAVIORS THAT ARE**
   - **THE MOST ECONOMICAL,**
   - **THE MOST PROBABLE,**
   - **THE MOST EFFICIENT,**
   - **THE BEST ADAPTED TO THE KNOWLEDGE ASSUMED…**
D. **AND CONSEQUENTLY TO FORESEE THEIR EFFECTS,**

E. **TO EXAMINE THE LOGICAL CONSISTENCY OF THESE MODELS AND THEIR COMPATIBILITY WITH ESTABLISHED RESULTS, WITH THE AID OF APPROPRIATE THEORETICAL CONCEPTS,**
F. **TO PUT THESE SPECULATIONS TO THE TEST IN SCIENTIFIC EXPERIMENTS,**
G. **AND FINALLY TO IMPROVE THE QUALITY OF LESSONS.**

- **DESPITE BEING MORE COMPLEX, THE MODELS OF SITUATIONS CAN BE ANALYZED MORE EASILY THAN THE PROBLEM THEY CONTAIN**

- **AND BECAUSE THEY ARE MORE "REALISTIC" AND MORE RATIONALLY ORGANIZED:**

1. THEY LEND THEMSELVES TO EXPERIMENTAL VERIFICATION,
2. THEY CAN FURNISH MEANS OF TEACHING THAT ARE MORE POWERFUL, MORE PRECISE AND MORE DEPENDABLE,
3. AND ABOVE ALL THEY MODEL COLLECTIVE BEHAVIORS RELATIVE TO THE INTENDED SHARED KNOWLEDGE, AND NOT JUST THE KNOWLEDGE AND BEHAVIOR OF EACH STUDENT AS AN ISOLATED SUBJECT.

- **CONSIDERING A CLASSICAL PROBLEM AND THE METHODS ASSOCIATED WITH IT AS A PARTICULAR KIND OF SITUATION PERMITS A BETTER ANALYSIS AND UNDERSTANDING OF ITS PROPERTIES.**
- **WHEN A CLASSICAL PROBLEM IS USED IN A CLASSROOM, IT IS ALWAYS WITHIN AN IMPLICIT SITUATION.**
- **IT IS IMPORTANT ALWAYS TO CHOOSE THE MOST EFFECTIVE SITUATION WHETHER IT REDUCES TO A CLASSICAL PROBLEM OR NOT**
- **REPLACEMENT OF A PROBLEM BY SOME SOPHISTICATED OR AMBITIOUS OR SIMPLY INNOVATIVE ACTIVITY SUGGESTED BY LOCAL OR SUPERFICIAL DIDACTICAL ANALYSIS IS A SERIOUS AND COMMON ERROR.**

2. CONSIDERATIONS ABOUT PSYCHOLOGY AND MATHEMATICS

- **A STUDY OF COGNITIVE PSYCHOLOGY CANNOT MODIFY GREATLY WHAT IT USES FROM ANOTHER DISCIPLINE, AND IN PARTICULAR THE ORGANIZATION AND VALIDITY OF THE KNOWLEDGE A SUBJECT IS SUPPOSED TO BE LEARNING.**
- **NEITHER CAN IT LOOK INTO THE CONDITIONS AND DEEP REASONS FOR THE USE OF THAT KNOWLEDGE.**
- **IN THE 70'S, STUDYING EXPERIMENTAL DESIGNS *FROM A MATHEMATICAL POINT OF VIEW* HAD NO PLACE AND ABOVE ALL NO STATUS IN PSYCHOLOGICAL RESEARCH (EXCEPT PERHAPS IN ERGONOMICS). RESULTS OF SUCH STUDIES WERE ACCEPTED OR REJECTED, BUT COULD NOT BE MODIFIED.**
- **BUT THE DEVELOPMENT OF THE STUDY OF THESE DESIGNS WAS JUST AS INDISPENSABLE FOR SCIENTIFIC RESEARCH IN PSYCHOLOGY AS FOR THAT OF TEACHING.**
- **SINCE PSYCHOLOGY DOES NOT INCLUDE WITHIN ITS FIELD OF STUDY THE *CONCEPTION* OF THE MATHEMATICAL AND MATERIAL CONDITIONS OF THE BEHAVIORS THAT IT STUDIES AND ABOVE ALL NOT THE CONTROL OF THOSE CONDITIONS,**
- **IT FOLLOWS THAT IT CANNOT PRODUCE ORIGINAL MODELS OF THE CONDITIONS, NOR PREDICT THEIR EFFECTS ON THE FORMS OF KNOWLEDGE THE CONDITIONS WILL PRODUCE.**

- **NOW, THE CONDITIONS ARE THE ONLY INSTRUMENTS AVAILABLE TO THE TEACHER FOR ACTING ON HIS STUDENTS.**
- **IT FOLLOWS THAT PSYCHOLOGY CAN NEITHER PREDICT NOR CONTROL THE INFLUENCE OF ITS RESULTS ON THE PRACTICE OF TEACHING.**
- **CONCLUSION: SCIENTIFIC STUDY OF DIDACTICAL PROCESSES NECESSARILY ESCAPES THE FIELD OF PSYCHOLOGY.**

When Psychology and *Didactique* interest themselves in the same system: Students – conditions (stimuli) – behaviors, their objects of study are different :

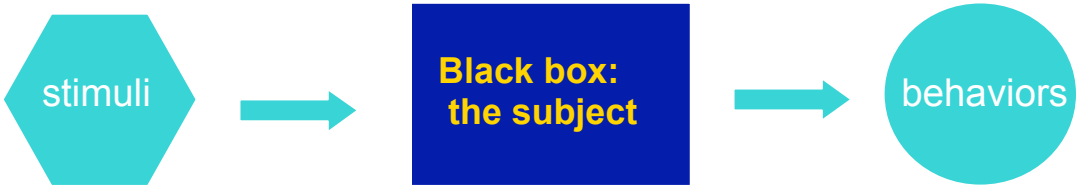
students for the former: Behaviors clarify the characteristics of the subject

conditions for the latter: Behaviors clarify the characteristics of the situation

Objects of study

Psychology

The psychological subject



Behaviors clarify the characteristics of the subject

Didactique

situations

Behaviors clarify the characteristics of the situation

- **PSYCHOLOGY HAS ALWAYS HAD A LOT OF INFLUENCE ON THE PRACTICES OF TEACHERS AND EVEN MORE ON THE DEMANDS OF THE PUBLIC.**
- **ALL OF US HERE KNOW THE DIVERSITY AND THE FORCE OF THAT INFLUENCE.**
- **THE CONSEQUENCES ARE VARIED, BOTH GOOD AND BAD, OBVIOUS AND HIDDEN…**

- **FOR INSTANCE: IN CONCENTRATING ON THE LEARNER AS SUBJECT, IT HAS CONTRIBUTED STRONGLY TO AFFIRMING THE IDEA THAT THE "IDEAL" DIDACTICAL RELATIONSHIP IS THAT OF AN INDIVIDUAL STUDENT WITH A SPECIFIC TUTOR.**

- **INDIVIDUAL TUTORING DISTORTS THE CONTENT AND ISN'T EVEN THE MOST EFFECTIVE WAY OF TEACHING A GIVEN CULTURAL OBJECT.**

- **ON ANOTHER FRONT, THE IDEA THAT PSYCHOLOGY IS THE ONLY LEGITIMATE DOMAIN FOR SCIENTIFIC STUDY OF TEACHING IS WIDESPREAD, EVEN IN THE SCIENTIFIC COMMUNITY.**

- **TO THE POINT WHERE A MINISTER OF EDUCATION COULD PRESENT COGNITIVE SCIENCE AND NEUROPHYSIOLOGY AS THE OFFICIAL SCIENCES TO BE TAUGHT TO TEACHERS (!)**

AND NOBODY PROTESTED IT!!

- **THIS COMMON CONCEPTION OF SCIENTIFIC RESEARCH ON TEACHING IS OVERSIMPLIFIED AND VERY INAPPROPRIATE. NONETHELESS IT KEEPS GROWING AND SPREADING.**

- **IT LED PEOPLE TO THINK THAT EXPERIMENTAL DESIGNS CONNECTED WITH MATHEMATICAL STUDIES COULDN'T BE SUBJECTED TO A STUDY THAT WAS AT ONCE THEORETICAL AND EXPERIMENTAL.**

- **SOME COURAGE WAS REQUIRED TO DECLARE THAT THE *STUDY OF THE CONDITIONS OF PRODUCTION AND TRANSMISSION OR RE-PRODUCTION OF MATHEMATICAL THOUGHT* WAS THE OBJECT OF A DIFFERENT SCIENCE.**

- **CLEARLY THIS SCIENCE CANNOT BE SUBSTITUTED FOR PSYCHOLOGY, NOR DOES IT DIMINISH ITS CREDIT**

- **ON THE CONTRARY, IT EXTENDS ITS INFLUENCE IN THE DIRECTION OF A NEW FIELD WHICH IT COULD NOT HAVE TAKEN ON SOLO.**

3. DIDACTICAL ENGINEERING OF MATHEMATICS

THE OBJECT OF *DIDACTICAL ENGINEERING* IS

- TO PRODUCE, ORGANIZE, AND TEST SITUATIONS AS INSTRUMENTS OF DIDACTICAL ACTION OF THE TEACHER

- TO MAKE EXPLICIT AND COMMUNICATE THE POSSIBLE OPTIONS

- TO JUSTIFY THE CHOICES AMONG THE OPTIONS BY ALL THE THEORETICAL AND EXPERIMENTAL MEANS OF *DIDACTIQUE*.

- **IT PROCEEDS AT EVERY STEP BY INQUIRING "WHY WOULD A STUDENT DO THAT?" AND LOOKING FOR ANSWERS THAT ARE CONVINCING AND/OR VERIFIABLE.**
- **THE CONDITIONS ACCEPTED ARE THOSE THAT ARE OPTIMAL IN PROBABILITY AND FOR COLLECTIONS OF STUDENTS**
- ***A CURRICULUM* IS AN ORDERED SEQUENCE OF MATHEMATICAL AND DIDACTICAL SITUATIONS THAT CAN CAUSE THE STUDENTS TO APPROPRIATE A COHERENT BODY OF MATHEMATICAL KNOWLEDGE.**
- ***TO CONCEIVE OF A CURRICULUM* IS TO CONCEIVE, FOR A GIVEN INSTITUTION, OF THE CONDITIONS FOR A SPECIFIC *GENESIS* OF MATHEMATICS AS IT IS PRACTICED IN ANOTHER INSTITUTION (MATHEMATICIANS).**
  - **AMONGST THE CONCEIVABLE GENESES, ONLY A FEW HAVE DIDACTICAL AND PRACTICAL VIRTUES APPROPRIATE FOR THIS PROJECT. THEY MAY DIFFER APPRECIABLY FROM THE HERITAGE OF THE PAST, FROM "INNOVATIVE" IMPROVISATIONS AND FROM THE NAÏVE EPISTEMOLOGICAL INFERENCES OF THE INSTITUTION OF SOURCE.**
- **THIS IMPLIES THAT THE SCHOLASTIC GENESIS MAY BE DIFFERENT – TRANSPOSED – FROM ITS SCIENTIFIC MODEL.**
- **OUR PRESENTATION COULD LEAD TO THE BELIEF THAT THE STUDY OF SITUATIONS AND ENGINEERING ARE DOWNSTREAM FROM THAT OF CLASSICAL PROBLEMS**
- **AND *A FORTIORI* DOWNSTREAM FROM MATHEMATICS ITSELF,**
- **AND THAT IT COULDN'T PUT EITHER OF THEM TO THE QUESTION.**
- **THAT'S NOT TRUE AT ALL!**
- **CONSIDERING SITUATIONS FOR INVENTION AND APPLICATION OF MATHEMATICS LEADS TO FUNDAMENTAL MATHEMATICAL, EPISTEMOLOGICAL, HISTORICAL AND DIDACTICAL REFLECTIONS.**
- **THESE STUDIES CAN ASK DEEP QUESTIONS AND OFTEN LEAD TO THE MODIFICATION OF THE ORGANIZATION OF THE DIDACTICAL**

TRANSPOSITIONS OF THE DISCIPLINE (THOUGH OBVIOUSLY NOT MODIFICATION OF THE VALIDITY OF THE THEOREMS).

- **IN ORDER TO IMPROVE TEACHING, CHOICES MUST BE MADE, OR NEW CONCEPTIONS CREATED, ABOUT THE ORGANIZATION OF WHOLE AREAS OF MATHEMATICS.**

- **FOR EXAMPLE, WE CAN CHOSE:**

  - **BETWEEN CONSTRUCTING DECIMAL NUMBERS DIRECTLY BY ADJOINING 1/10 AS AN OPERATOR OR FOLLOWING THE HISTORICAL ORDER AND CONSTRUCTING THEM AS A RESTRICTION OF THE FIELD OF FRACTIONS,**

  - **BETWEEN CONSTRUCTING STATISTICS AS AN APPLICATION OF PROBABILITY OR DIRECTLY AS A MEASURE OF EVENTS (DEPENDING ON THE CHOICE, HYPOTHESIS TESTING IS NEAR THE BEGINNING OR NEAR THE END OF THE SEQUENCE),**

  - **BETWEEN STUDYING GEOMETRY BY EXPLORING PROPERTIES OF FIGURES OR THOSE OF TRANSFORMATIONS OF THE PLANE AND THEIR INVARIANTS.**

- **MORE GENERALLY, THERE MAY BE A CHOICE BETWEEN A SYSTEMATIC — AXIOMATIC OR HISTORICAL — CONSTRUCTION AND A MORE ERRATIC PRESENTATION.**

- **THE USUAL ASSUMPTION IS THAT EVERY OPTION SHOULD FIRST BE DETERMINED WITHIN THE MATHEMATICS ITSELF, IN OTHER WORDS, WITHOUT REFERENCE TO ANY SPECIFIC DIDACTICAL ISSUES,**

- **BUT THESE DETERMINATIONS ARE BASED ON IMPLICIT, SPONTANEOUS DIDACTICAL AND EPISTEMOLOGICAL ASSUMPTIONS THAT RADICALLY LIMIT THE POSSIBILITIES OF REALIZING THEM.**

- **THE REQUIREMENTS OF DIDACTICAL ENGINEERING SHOULD THUS BE TAKEN INTO CONSIDERATION AT THE MOMENT OF THE ORGANIZATION OF THE AREA TO BE TAUGHT, AT THE SAME TIME AS THE MATHEMATICAL NECESSITIES. WITHOUT THAT, NOVEL DIDACTICAL SUGGESTIONS HAVE NO CHANCE OF PROVING THEMSELVES TO BE ANY BETTER THAN THE TRADITIONAL APPROACHES.**

- **THE WEAK LINK IS THE ACCEPTANCE BY MATHEMATICIANS OF THE RESULTS OF THIS WORK.**

- **THE STUDY OF MATHEMATICAL SITUATIONS AND DIDACTICAL ENGINEERING ARE THE HEART OF *DIDACTIQUE*, AND THEY SHOULD BE THE ACTIVITY OF A MATHEMATICIAN PRIMARILY ADDRESSING HIS COMMUNITY.**

•DIDACTICS BECOMES EXPERIMENTAL SCIENCE "THE SCIENCE THAT STUDIES THE CONDITIONS FOR TEACHING THE MATHEMATICAL KNOWLEDGE THAT IS USEFUL TO SOCIETIES", IF IT CAN BE ACCOMPANIED BY SPECIFIC METHODS FOR CONFRONTING ITS STATEMENTS WITH REALITY.

•IT CAN BE OBSERVED IN ANY CASE THAT THE STUDY OF THE ENGINEERING OF THE DESIGNS USED IN EXPERIMENTS ON COGNITIVE PSYCHOLOGY IS ESSENTIAL.

4. OBSERVATION OF TEACHING

•THE "GRAND *DIDACTIQUE"* OF COMENIUS LEAVES NO PLACE FOR CONTINGENCY AND HENCE FOR EXPERIMENTATION.

•MOREOVER, EVERYTHING DISCOURAGES EXPERIMENTATION:
- THE COMPLEXITY OF DIDACTICAL RELATIONS AND OF THEIR COMPONENTS,
- THE ABUNDANCE OF PERTINENT EMPIRICAL KNOWLEDGE IN EVERY HUMAN
- AND THE NUMBER OF DISCIPLINES THAT CAN INTERVENE IN STUDYING THEM.

•OBSERVATION TOO, RUNS INTO ENORMOUS HIDDEN DIFFICULTIES, PRACTICAL AND THEORETICAL.

•THAT IS WHAT WE FIRST BEGAN TO STUDY, STARTING AT THE END OF THE 60'S.

•THE ISSUE WAS THE OBSERVATION OF THE TEACHING ITSELF,

•BUT NOT OF ANY OF ITS COMPONENTS IN PARTICULAR.

•THE ORGANIZATION OF THE MINIMAL CONDITIONS FOR OBSERVATION LED TO THE CREATION OF A SPECIFIC TEACHING ESTABLISHMENT: MICHELET SCHOOL, ASSOCIATED WITH A CENTER FOR OBSERVATION ANDE RESEARCH ON MATHEMATICS EDUCATION (THE COREM).

THE DESIGN OF THE COREM WAS CONCEIVED ON THE SAME PRINCIPLES AS THOSE USED IN THE ENGINEERING OF SITUATIONS:
A) CONSIDER THE SET OBSERVER/OBSERVED TO BE A SYSTEM, CO-RESPONSIBLE FOR THE WORK AND SCHOLASTIC RESULTS AS WELL AS THE FUNCTIONING OF THE RESEARCH.
B) REDISTRIBUTE THE RESPONSIBILITIES AND MAKE SURE THAT AT ALL LEVELS OF DECISION THE REGULATIONS AND CONTROL NECESSARY FOR MAINTAINING THE MINIMAL CONDITIONS ARE PRESENT.
C) HAVE THE MEANS AND TIME NECESSARY TO CORRECT UNFORTUNATE DECISIONS.

D) MODIFY THE DESIGN ONLY TO *INSURE THAT IT DISPLAYS THE INFORMATION UNDER STUDY AND ITS ACTUAL USE.*

EXAMPLE

Two teachers together prepare two successive lessons to be given to the same class.
One of them carries out the first lesson in the absence of the second, who is to carry out the next lesson.
A recording is made of the information that the second asks of the first and of what the first offers the second.
A recording is made of the discussion after the second class.
The observers compare the opinion of the teachers with their own hypotheses and with the recordings of the lessons.

- **THE FIRST DIFFICULTIES FOR COREM:**
- **RECONCILING THE NEW DISTRIBUTION OF POWERS OF DECISION IN THE SYSTEM OF OBSERVATION WITH LEGISLATION AND LEGITIMATE HABITUAL PRACTICES,**
- **ARRANGING FOR THE ADMINISTRATION TO SUPERVISE THE WHOLE SYSTEM AT ALL TIMES, AND FOR THE PUBLIC TO REGARD THE SCHOOL AS AN "ORDINARY" ESTABLISHMENT – BOTH NECESSARY TO GUARANTEE ITS SURVIVAL,**
- **AVOIDING THE HABITUAL DEMANDS RESULTING FROM NAÏVE CONCEPTIONS OF RESEARCH: COMPETITION, OSTENTATION, PREMATURE DIFFUSION, INNOVATION,…**
- **GAINING THE CONFIDENCE OF THE INSTITUTIONS IN ORDER TO REALIZE THIS OBSERVATION DESIGN**
- **ONLY THE IREM'S MADE THIS MIRACLE POSSIBLE.**
- **THE SLOW AND CONTROLLED EVOLUTION OF THE OBJECTS OF OBSERVATION:**
- **DURING 12 YEARS ('70 – '82), STUDIES WERE LIMITED TO MATHEMATICAL SITUATIONS ONLY – THE TEACHER AND THE OBSERVER SIDE BY SIDE OBSERVED THE MATHEMATICAL SITUATIONS AND THE STUDENTS' REACTIONS.**
- **LATER (82-95) STUDIES WERE EXTENDED TO MICRO-DIDACTICAL SITUATIONS: OBSERVATION INCLUDED THE TEACHER AS AN OBSERVED SUBJECT.**
- **WHY? WHY WERE THE STEPS SO LONG?**
  - **EACH REQUIRED AN EVOLUTION AND ADAPTATION OF THE RESEARCHERS, OF THE TEACHERS AND OF THE KNOWLEDGE THEY USED.**

•IMPORTANT REMARK: ONLY GENERALIZABLE BEHAVIORS WERE OF INTEREST, NOT SINGULARITIES OF THE PERSON TEACHING.

•FOR EXAMPLE, AN ERROR MADE BY THE TEACHER IS OF INTEREST TO THE RESEARCHER ONLY IF THE RESEARCHER CAN SEE IN IT THE PROTOTYPE OF A PHENOMENON THAT OTHERS MAY REPRODUCE IN FAIRLY FREQUENT CIRCUMSTANCES.

•SATISFYING THE CONDITIONS FOR EXISTENCE AND STABILITY OF THE OBSERVER/OBSERVED SYSTEM LED TO THE ELABORATION OF AN APPROPRIATE AND ORIGINAL METHODOLOGY.

•AMONG OTHERS:

•- "A PRIORI" AND "A POSTERIORI" ANALYSES

•- GENERALIZED USE OF NEW AND MORE APPROPRIATE STATISTICAL METHODS.

•THE TECHNIQUE OF COMPARING EFFORTS WHEN THE RESULTS ARE THE SAME.

•(THIS TECHNIQUE OPENED UP A FAR RICHER ROUTE THAN COMPARISONS BASED SOLELY ON THE COST OF SUCCESS, WHICH HAVE FAR TOO MUCH INFLUENCE ON THE THEORETICAL CONCEPTIONS)

•- AND SO ON

•FROM OBSERVATION TO EXPERIMENTS…

•WITHIN THIS WELL STABILIZED OBSERVATION DESIGN, THE THEORETICAL APPROACH ENABLED US TO CARRY OUT EXPERIMENTS WITH A RIGOROUS PROTOCOL,

•FIRST SMALL EXPERIMENTS ON DIDACTICAL ERGONOMY…

•AND LATER SOME OTHERS:

- ON THE EFFECTS OF THE DIDACTICAL CONTRACT,
- ON THE LIMITATIONS OF CONSTRUCTIVISM,
- ON THE LIMITATIONS OF THE THEORY OF DEVELOPMENTAL STAGES.
- ETC.

Now I can give a better explanation of my past relationship with PME:

KARLSRUHE 1976 : PME

- **EFRAÏM FISHBEIN, GÉRARD VERGNAUD, AND SEVERAL OTHERS REQUESTED THAT ICMI CREATE A PERMANENT SUBCOMMISSION ON SCIENTIFIC RESEARCH ON MATHEMATICAL EDUCATION.**
- **TRADITIONALLY, MATHEMATICIANS' REFLECTIONS HAD BEEN PLACED UNDER THE AEGIS OF PHILOSOPHY (POINCARÉ, CAVAILLÈS) OR PSYCHOLOGY (HADAMARD, POLYA : HEURISTICS).**
- **ONLY PSYCHOLOGY SEEMED CAPABLE OF SCIENTIFICALLY DEMONSTRATING AND IMPROVING THE INTENDED DIDACTICAL PROPOSITIONS FOR MODERNIZING THE DISCOURSE AND USE OF MATHEMATICS IN TEACHING.**
- **RESEARCH ON PEDAGOGY OR METHODOLOGY OR SOCIOLOGY, ETC. WAS WELCOME, BUT ONLY IN THE GENERAL CONGRESS ALONG WITH ALL TYPES OF PRESENTATIONS**
- **THE COMMISSION WAS THUS CREATED UNDER THE AEGIS OF PSYCHOLOGY ALONE: PME WAS BORN OF THE COMBINED SUCCESSES OF THE STRUCTURALISM, GENETIC EPISTEMOLOGY AND AMERICAN BEHAVIORAL PSYCHOLOGY.**
- **THIS PROVED TO BE A PRODUCTIVE CHOICE: PME HAS ACCOMPLISHED A LOT AND TODAY ENJOYS A LARGE AND WELL-EARNED SUCCESS.**
- **DIRECT EXPERIMENTAL SCIENTIFIC STUDY OF THE ACTS OF TEACHING AND LEARNING SEEMED AT THAT TIME UNPROMISING AND TOO COMPLEX, BECAUSE THE BELIEF AT THE TIME WAS THAT IT WOULD NECESSITATE FIRST, AND INDEPENDENTLY, BETTER UNDERSTANDING THE DIFFERENT COMPONENTS OF THE DIDACTICAL ACT: STUDENTS, TEACHERS, INSTITUTIONS, MATERIAL, ETC**
- **A GREAT EFFORT HAS BEEN AND CONTINUES TO BE REQUIRED TO INTRODUCE AN ENTIRE NEW SCIENTIFIC (I.E., THEORETICAL AND EXPERIMENTAL) BRANCH THAT**

-TAKES ITS PLACE AMONG THE MATHEMATICAL SCIENCES
- HAS AS ITS GOAL KNOWLEDGE OF THE PRACTICE OF TEACHING MATHEMATICS
-AND ALLOWS THE USE AND SURVEILLANCE OF KNOWLEDGE IMPORTED FROM OTHER DOMAINS.

IT HAS RESULTED IN *DIDACTIQUE* OF MATHEMATICS

Time out, for a technical detail : we are in the process of changing the English title from Didactique to Didactics, (like Linguistics, Economics etc.)

6. CONCLUSION

- **I WOULD HAVE LIKED TO SHOW HOW OUR WORK AND YOURS COULD COMBINE TO COUNTER SOME OF THE CURRENT HEAVY TENDENCIES IN OUR EDUCATIONAL SYSTEMS:**
- **A TENDENCY TOWARDS TOTAL INDIVIDUALIZATION OF TEACHING,**
- **AN EXCESSIVELY "PSYCHOLOGICAL" AND NEURO-SCIENTIFIC CONCEPTION OF SCHOLASTIC KNOWLEDGE CONFUSED WITH SKILL AND UNDERSTANDING**
- **A SENSELESS PRETENSE OF TREATING TEACHING AS IF IT WERE COMMERCIAL DISTRIBUTION, WITH THE CONSEQUENCE OF BARBAROUS USES OF SCHOLASTIC EVALUATIONS**
- **A THOUGHTLESS USE OF POPULAR OPINION AND "EXTREMIST" REASONING.**
- **IT IS NOT REASONABLE TO WISH TO TRANSFORM THE TEACHING THAT IS PRACTICED ON THE BASIS OF NAÏVE INFERENCES OR SUPERFICIAL EXPERIMENTS.**
- **NEW PRACTICES SHOULD NOT BE PROPOSED BEYOND THE EXTENT TO WHICH THEY CAN BE UNDERSTOOD AND MANAGED BY THE SYSTEM. THAT EXTENT DEPENDS ON ITS CULTURE.**
- **DIDACTICAL SCIENCE AND CULTURE MUST THEREFORE BE ADVANCED WITHOUT REQUIRING THAT THERE BE IMMEDIATE APPLICATIONS IN PRACTICE.**
- **IN *DIDACTIQUE*, AS IN OTHER SCIENCES, IMPATIENCE SEEMS TO ME TO BE ONE OF THE PRIME CAUSES OF TROUBLE.**

7. GRATITUDE

- **TO PME FOR HAVING DONE ME THE HONOR OF INVITING ME AND ACCEPTING ME INTO YOUR MIDST**
- **TO MY TRANSLATOR AND COLLABORATOR, GINGER WARFIELD, ON WHOM I HAVE IMPOSED TOO MANY HOURS OF WORK AND BELATED CORRECTIONS (WHICH SHE ENJOYED ! [GW])**
- **AND TO ALL THE PARTICIPANTS FOR HAVING KINDLY LISTENED TO MY PROPOSITIONS AND PUT UP WITH MY ACCENTED ENGLISH.**
- **MY THANKS TO YOU ALL.**