# Mathematical Activity and Evaluation

A Study in the Theory of Situations



- Guy Brousseau
- Théorie des Situations -- late 60's
- Felix Klein Award 2003 from the International Commission on Mathematics Instruction (ICMI), which was founded by Felix Klein in 1907.



- The first Felix Klein Award of the Internal Commission on Mathematical Instruction (ICMI) is awarded to Professor Guy Brousseau. This distinction recognizes the essential contribution Guy Brousseau has given to the development of mathematics education as a scientific field of research, through his theoretical and experimental work over four decades, and to the sustained effort he has made throughout his professional life to apply the fruits of his research to the mathematics education of both students and teachers.



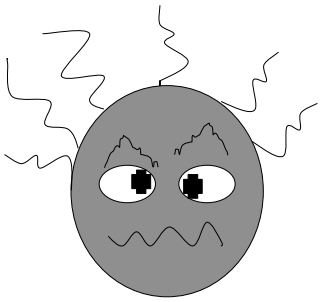
- Translations began in 1991
- Theory of Didactical Situations in Mathematics (Kluwer 1997)



- Savoir ⟶ to Know
- Connaître ⟶ to Know
- Savoir = Connaissance ???
- NON!!!



- Savoir ⟶ Knowledge
- Savoirs ⟶ Knowledge
- Connaissance ⟶ Knowledge
- Connaissances ⟶ Knowledge



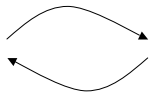


- Know-how
- S-knowledge
- C-knowledge
- Or sometimes just knowledge



- Different classes of situations needed different models:
- 1) Those in which the student ought to be able to identify, formulate and discuss the knowledge in question: savoirs.
- 2) Those in which the knowledge is manifested by actions, but not necessarily something the student is even conscious of knowing: connaissances.



- Savoirs  Connaissances



- Decisions in uncertain circumstances are made using connaissances.
- Savoirs are used as reference points.
- Savoirs and connaissances: necessary and complementary, and learned by different processes in different circumstances.



- EXAMPLES:1. FUNCTION from the point of view of the mathematical community
- 18th century: a connaissance
- A century later: a savoir — defined, categorized, a reference point.

- 2. Proportions:
- "A father is 5 times as old as his son. The son is 7 years old. How old is the father?"
- "The father is 35, because 5 x 7 = 35" is a savoir for a 3rd grader.
- Statements like
  - "Not all fathers are 5 times as old as their sons"
  - "No father is always 5 times as old as his son",etc.

  may be connaissances for some children, but the teachers can neither teach nor require them for all of the children.



- He could nonetheless ask:
  "How old will the father be when his son is 40?"
- Some students may well answer: 5 x 4 = 200 as a result of an implicit didactical contract: they apply the most familiar operation, knowing (connaissant) full well that the answer is unrealistic. In effect "No human being lives 200 years" is probably a savoir in their everyday life, but in school it's just a connaissance. Only for the strong students, the ones who can present this objection to the teacher, does it have the status of savoir.



- Theorem ⟷ Savoir
- Conjecture ⟷ Connaissance



- Problems are posed to develop connaissances using existing savoirs
- Exercises are given to solidify and streamline savoirs and their uses.



- Mathematics doesn't burst into existence as a series of theorems with slick proofs.



- *Didactique (*Didactics), and in particular research in the Theory of Situations, studies ways in which a small community of students can adapt itself to mathematics by developing and using a wide variety of connaissances.
- Hence also a study of connaissances and their relationship to teachers' and students' decisions.

Objects of a low level in Bloom's cognitive taxonomy (reproducing a definition, applying an algorithm…) ⟷ Savoirs

Objects of high level (solving a new problem, synthesizing…) ⟷ Connaissances



- Savoirs ⇝ Skills
- Connaissances ⇝ Understanding



# The Theory of Mathematical Situations



- The Theory of Mathematical Situations was introduced in order to study the designs of psychological experimentation. It is based on modeling the interactions between a group of people (considered as the agents) and a *milieu* — the parts of the environment that lead the agents to manifest and learn the specific behaviors of a precise piece of mathematical knowledge.
- It enables us to examine and compare facts in the realm of mathematics that come from the domains of its epistemology, of its history, of its pedagogy, of psychology or even of sociology.



# The Evaluation of Knowledge



- Society delegates to teachers the responsibility for acculturating students to its practices and to its knowledge, sometimes even for making them instruments for its projects.
- It does so on faith in a possible distribution of responsibilities, assumed to be verifiable.
- The node point of this contract is the determination of knowledge "to be acquired" and of knowledge effectively "acquired" by the students.

- Traditionally this determination was implicit and entrusted to the practices of the academic community or the teacher (who assigned grades at his convenience.) It has become more and more explicit and contractual by means of more and more precise texts and more and more formal *evaluations*.
- The "rationalization" of these new contracts produced paradoxical constraints.
- The theoretical study and observation of these constraints led to the development of a theory of didactical situations (in mathematics) complementary to the theory of mathematical situations.



- Institutional evaluations must satisfy difficult conditions: they must be given to large groups and corrected quickly, they must make it possible to identify the knowledge to be taught, etc.
- So each item is assumed to evaluate *a* piece of knowledge (as an objective or as an acquisition).
- The student must produce this piece of knowledge in a very narrow context and independently of its functioning.
- Thus it is mainly called forth as an isolated skill, as a reference, thus as a text, or at best an exercise, because it is very difficult to use problems to make evaluations.



- Only skills can be evaluated by these methods.
- Understanding cannot be "evaluated" by these means (by definition).
- Now, it represents a result of teaching that is important — it's the one we're after when we assign problems — and is necessary for the comprehension required for further learning.
- The use teachers make of these evaluations depends on their capacity to interpret them and to make didactical decisions based on them and to convince others of the validity of those decisions.



Studies we made in the early 70's demonstrated:

1. That teachers could predict their students' test results at any level of significance *only* for questions that called for skills.
2. That this was the only form of knowledge for which they could easily prepare characteristic questions and exercises.



3. That in general teachers' response to a failure was:
   - a presentation of the correct answer, or
   - a repetition of the original explanation followed by the same question,
   - a rapid search for some specific information to reduce the uncertainty or,
   - a subdivision of the skill into "simpler" — i.e. more closed — skills, etc.

Numerous processes of subdivision were all based on formal analysis of the mathematical texts and skills.



4. That at the time of relaunching the whole process, the knowledge the students had already acquired was ignored or considered to be a supplementary difficulty.
5. That the students' difficulties were perceived by the teachers only in terms of decisions about how to remediate them.

Various other studies showed:

- That institutional evaluations concentrated attention on skills, forms of knowledge the closest to texts
- That they led the teachers

  - to look for ways to improve the results by closed methods, which reduce the role of understanding

  - to use the logic of the evaluation (pedagogy by objectives) and even its form (individual worksheets) as means of teaching.



- The studies also showed that teaching the texts alone doesn't give most students the capacity to solve problems.
- Inversely, that drill in solving problems produced only an uncertain improvement in the results of institutional evaluations
- And that the teaching problem solving methods does not resolve the difficulties.



- This work led us to formulate the hypothesis that:
- Institutional evaluations cannot suggest relevant corrections in the course of teaching, at least not with classical didactical practices.
- The long-term effects of such a use of evaluations was predictable: our study was published in 1978 and events continue to confirm its predictions.



- The work we just cited led us to re-pose the problem from the point of view of a mathematical activity, simultaneously developing skills and understanding in their reciprocal relationship, and to look for other means of evaluation for the learning phases.



# Abusive Uses of Evaluations



- Institutional evaluation of skills and understanding is necessary and legitimate. It is very important to have information on the behaviors of students and teachers.
- The critical questions are: who is evaluating what? How are they going to use it? What instruments will be used? Validated by what social processes and what scientific knowledge?
- All partners in education should obtain the evaluations they need and should be warned of and protected from their unconsidered usage.
- But abuses of evaluation can originate from any of these factors and any of the partners.

- Most of these abuses result from our ignorance and our illusion of understanding teaching well.
- Social treatment of knowledge and its transmission is a practice as old as humanity, and of an unparalleled degree of complexity. But there is very little that we know with any certainty about its spontaneous modes of regulation.
- For sure, the objects and the means of institutional evaluation are determined by the disciplinary, epistemological and didactical cultures of the evaluators.



- But "transparent" evaluations are based on community cultures that are older, more primitive and more drastic. And these are the evaluations that currently take over as a last resort.
- Incautiously subjecting a process that is so complex and so essential to a radical innocence can wipe out centuries of efforts.
- Under the cover of democracy, naïve evaluation is currently invading the very heart of didactical activity.
- Society ought to protect certain activities that are essential to it from barbaric evaluations and cultural or scientific vandalism.



- We will restrict ourselves to pointing out more modest, or rather more insidious, abuses: the immoderate use of evaluations in the course of the learning and teaching of mathematics.
- Very little systematic and theoretical study has been given to the evaluation of teaching. It is constantly used and scrutinized, but there are no indications of the limiting conditions of its use.
- The Theory of Situations offers here an example of an embryo of this type of study.



# Immediate consequences of the abusive use of evaluations



- In the past, society was satisfied if it knew that teachers had covered a body of knowledge. The learning and use of it were the responsibility of the student.
- Today society requires a validation of the "results" of each step of teaching by evaluating the "knowledge acquired" by the student.
- This requirement is based on the following hypothesis:
- Each step of teaching leads towards an "end knowledge" which must be:



- Identifiable as part of the reference text (but only savoirs and skills),
- Observable as behaviors of the students (in what conditions?),
- Teachable — there should be at least one standard method for obtaining it (actually or theoretically?)
- Sufficient to allow the next step to be taught (to begin studying it? To insure its success?),
- Minimal — that is, necessary for the next step (…

- This hypothesis is based on a particular epistemological conception in which all knowledge can be acquired by a known combination of previously acquired skills (savoirs), as in a standard mathematical exposition.
- But the resulting contract opens up a recursivity. Any step that results in a failure must be fragmented into sub-steps.
- There is no theoretical limit to this fragmentation, only a practical limit.



- Now, the epistemological and psychological foundations of this hypothesis are erroneous and the conditions of learning and institutional evaluation are contradictory.
- Furthermore, if the role of understanding is ignored, the possibilities for the students and for the work of the teachers are badly underestimated.
- Institutional evaluation of teaching is, like many social practices, based on fiction and fantasy.



- The focus on topics leads teachers to choose to teach the topics directly, to reduce learning situations to a relationship between the learner and the specific topic and limit the role of understanding.
- The errors and uncertainties that naturally occur in the course of learning are labeled in classical evaluation as failures in a teaching sequence.
- This calls forth one of the classical didactical decisions that ignore the role of the understanding at work and, in the course of teaching, sterilize it.



- The "report of failure" obligates the teachers
  - 1. to repeat all or part of the sequence and usually to split it up, by repetition or decomposition of the knowledge — hence to a lengthening of the teaching time,
  - 2. To increase the individual actions on students "having trouble", thus to discriminations that slow the teaching,
  - 3. To chose specific methods, such as individual worksheets that reproduce the evaluations,



  - 4. Above all to chose the most closed situations and to limit as much as possible the recourse to understanding
    - To avoid the dispersion of errors
    - And too many individual interventions
    - To reattach the results
    - To facilitate common didactical decisions, etc.
- Closed situations seem more certain and more rapid, but in the long run they extend the length of teaching time and augment failures.



- Individualization of teaching tends to isolate each student in a learning bubble that is all the more airtight in that the idea of cooperation doesn't mesh with some uses of evaluation or some individualistic ideologies.
- This leads the teacher to multiple repetitions of the same "private" intervention (tutoring each student).
- Focusing on individuals contributes to the disappearance, from the students' eye view, of the practical, immediate and social reasons for knowing and learning.
- Mathematical activity winds up appearing to be an solitary delight, although it is a cultural and social activity that can be interesting at all levels.

# Long term consequences of the abusive use of evaluations



- Closure is reinforced by an implicit clause of the didactical contract dealing with tests given in class that tends to emerge:
- The right of the teacher to "evaluate" knowledge that he has not taught explicitly is contested by the students and their parents.
- It becomes difficult for him to pose problems, not only on tests but also in the course of teaching.
- The higher the stakes on the evaluation, the more vigorously this law is invoked.
- This clause contributes further to the elimination of possibilities for understanding to function.



- This rule that makes the sufficiency of prior knowledge a didactical obligation exerts a strong influence on the order of lessons.
- It restricts the organization of the curriculum to systematic "rational" constructions: from part to whole, from concrete to abstract or vice versa, and restricts the organization of mathematics to axiomatic presentations.
- These convenient didactical organizations have nothing to do with the spontaneous genesis of mathematical knowledge, nor with its functioning.



- We have demonstrated that newly learned skills are more difficult to use, and consequently more difficult to mobilize when they are stripped of the understanding that permits their activation.
- Skills that result from the subdivision of objectives are about as difficult to teach as the failed original skills — often even more so.
- Beyond a certain limit, the didactical cures induced by the use of these evaluations thus no longer produce better results. On the contrary, they cost more time.



A different recursive process then sets in:

- The objectives are not achieved
- Teachers show that they want to achieve them by closing off situations
- Which in turn makes it more difficult to acquire further understanding and increases the disappointment.
- Teachers of step *n* request more time or else reduce the objectives while those of step *n+1* want to increase them
- Whatever the response, the level sinks.



- The disappearance of understanding annihilates the steps in the didactical transposition whose object is to make the students learn mathematics by getting them to take part in an activity that simulates the way professionals use and produce it:

  A) the end result of a mathematician's activities (of his knowledge and skills) is an impersonal, objective, general mathematical text, detached from conditions and history (decontextualization)

B) The teacher needs to make this text correspond to a situation (for instance by turning it into a problem) in which the student can invest his knowledge and his activities in order himself to produce similar knowledge, by its use, its structure, its meaning, the questions it poses … and not just by its form (recontextualization).

C) This knowledge in its turn should come to be expressed and recognized as a scholarly text (redecontextualized).



# Conclusions



- Evaluation is expected to be, and is used as, an instrument for solidifying educational relationships.
- But the conceptions on which it is based are inadequate.
- Making it a means of managing teaching presents enormous dangers because, like all contradictory systems, it permits all conclusions, even the most excessive.
- To abbreviate my remarks into a slightly dubious metaphor, coachmen want to whip locomotives to make them go faster.



The solution is clear:

- Develop better scientific knowledge about didactical phenomena we have been acquainted with since the beginning of humanity but about which we know very little.
- And to do that, develop fundamental research specific to didactics (here the didactics of mathematics): not one of the classical branches of research, no matter how useful it may be, can lay claim to being able to handle this category of phenomena (this holds for mathematics and psychology)



- Improve the institutional system for assessment of the problems of elaboration, management and diffusion of knowledge
- Not believe that processes this complex can be improved with a few drastic administrative measures
- Not make school the instrument of pitiful but very harmful political coups.