On the Theory of c-Systems

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Contents

§1. Introduction

In our discussion of the Selberg trace formula centering on the contribution to the trace arising from the continuous spectrum (cf. [1-(c)]), we formulated a conjecture that is of essential importance for the theory. We shall not stop to recall its statement here since this would be a little bit involved. Instead, the purpose of the present note is to approach the conjecture abstractly via the artifice of Detroit families (cf. [1-(a)]). While we have no proof in general of the conjecture, we shall nevertheless be able to say some things that seem to be of intrinsic interest and which, we feel, may well lead eventually to its proof. The main result is a reduction theorem. This sets the stage for induction arguments, one consequence along these lines being what appears to be a fairly substantial simplification in the statement of the conjecture itself, detailed at the very end of the paper.

§2. Two-Variable Detroit Families

The notion of a Detroit family is developed in [1-(a)], to which we refer the reader for an explanation of the undefined symbols and terms appearing below. This said, our immediate objective will be to systematize the considerations of [1-(b), §5], recasting matters in abstract terms.

So let (V, Φ) be a geometric g.r.s. – then by a two-variable Detroit family ϕ we understand a collection of C^∞ functions

$$\phi_C : \sqrt{-1}V \times \sqrt{-1}V \to \mathbf{C}$$

with the property that for every pair of C^∞ functions

$$\mu, \nu : \sqrt{-1}V \to \sqrt{-1}V$$

satisfying the condition

$$H = \mu(H) + \nu(H),$$

the collection of assignments

$$H \to \phi_C(\mu(H), \nu(H))$$

constitutes a Detroit family.

Now put

$$\Phi_\Delta = \{(\lambda, \lambda) \in V \times V : \lambda \in \Phi\}.$$

Because it does not span, Φ_Δ is not, strictly speaking, a geometric g.r.s. in $V \times V$; however, Φ_Δ is a geometric g.r.s. in

$$V_\Delta = \{(H, H) \in V \times V : H \in V\}.$$

Actually, since the proofs really take place in V_Δ, the fact that Φ_Δ fails to span is not at all serious.

For example, the definition of a chamber in $V \times V$ per Φ_Δ is clear enough. Let $C\Delta C$ be one such and let C_Δ be the corresponding chamber in V_Δ. Setting

$$V_0 = \{(H_0, -H_0) \in V \times V : H_0 \in V\},$$

we have

$$C\Delta C = C_\Delta + V_0$$

and Lemma 3.5 in [1-(a)] would then read "span $(F) = V_\Delta$". On the other hand, if

$$\pi : V \times V \to V$$

is the projection, then

$$\sqrt{2}\pi : V_\Delta \to V$$

is an isometry sending Φ_Δ to $\sqrt{2}\Phi$, which has the same chambers as Φ itself, thus to C_Δ there corresponds a chamber C per Φ. Symbolically:

$$C \Delta C \leftrightarrow C_\Delta \leftrightarrow C.$$

Theorem 2.1. *Let ϕ be a collection of C^∞ functions*

$$\phi_C : \sqrt{-1}V \times \sqrt{-1}V \to \mathbf{C}.$$

Then the following conditions are equivalent:

(1) *ϕ is a two-variable Detroit family.*
(2) *ϕ is a Φ_Δ-Detroit family on $\sqrt{-1}(V \times V)$.*
(3) *$\forall H_\nabla \in \sqrt{-1}V_0$, $\phi(H_\nabla + ?)$ is a Φ_Δ-Detroit family on $\sqrt{-1}V_\Delta$.*

Suppose that we are given two triples

$$\begin{cases} C'\Delta C' \leftrightarrow C'_\Delta \leftrightarrow C' \\ C''\Delta C'' \leftrightarrow C''_\Delta \leftrightarrow C''. \end{cases}$$

Then it is clear that
$C'\Delta C'$, $C''\Delta C''$ are adjacent at (λ, λ) iff
C'_Δ, C''_Δ are adjacent at (λ, λ) iff
C', C'' are adjacent at λ.
The contention of the theorem therefore amounts to claiming that the following conditions are equivalent:

(1) If

$$\mu, \nu : \sqrt{-1}V \to \sqrt{-1}V$$

are C^∞ functions such that

$$H = \mu(H) + \nu(H),$$

then $\forall H \in \lambda^\perp$,

$$\phi_{C'}(\mu(H), \nu(H)) = \phi_{C''}(\mu(H), \nu(H)).$$

(2) If

$$\begin{cases} H \in \sqrt{-1}(V \times V) \\ H \in (\lambda, \lambda)^{\perp}, \end{cases}$$

then

$$\phi_{C'}(H) = \phi_{C''}(H).$$

(3) If

$$H_{\nabla} \in \sqrt{-1}V_0, \quad \begin{cases} H_{\Delta} \in \sqrt{-1}V_{\Delta} \\ H_{\Delta} \in (\lambda, \lambda)^{\perp}, \end{cases}$$

then

$$\phi_{C'}(H_{\nabla} + H_{\Delta}) = \phi_{C''}(H_{\nabla} + H_{\Delta}).$$

When stated this way, (2) and (3) are obviously equivalent. Indeed, the orthocomplement of (λ, λ) in $V \times V$ is the direct sum of V_0 and the orthocomplement of (λ, λ) in V_{Δ}. The fact that (2) $\Rightarrow$ (1) is also plain. To finish, we shall prove that (1) $\Rightarrow$ (3). Assuming that $H_{\Delta} \in \sqrt{-1}V_{\Delta}$, write

$$H_{\Delta} = (H, H).$$

Then

$$H_{\Delta} \in (\lambda, \lambda)^{\perp} \Leftrightarrow H \in \lambda^{\perp}.$$

Next, write

$$H_{\nabla} = (H_0, -H_0)$$

and define

$$\begin{cases} \mu(H) & = \frac{1}{2}H + H_0 \\ \nu(H) & = \frac{1}{2}H - H_0. \end{cases}$$

Then

$$H = \mu(H) + \nu(H).$$

Since

$$H \in \lambda^{\perp} \Rightarrow 2H \in \lambda^{\perp},$$

we have

$$\phi_{C'}(\mu(2H), \nu(2H)) = \phi_{C''}(\mu(2H), \nu(2H))$$

or still

$$\phi_{C'}(H + H_0, H - H_0) = \phi_{C''}(H + H_0, H - H_0)$$

or still

$$\phi_{C'}(H_{\nabla} + H_{\Delta}) = \phi_{C''}(H_{\nabla} + H_{\Delta}),$$

as desired.

Example. Let $\mathbf{H} = \{H_C\}$ be a Φ-orthogonal set of points in V – then, as is well-known, the functions

$$H \rightarrow \exp[(H, H_C)]$$

form a Detroit family $e^{\mathbf{H}}$. And, the theory supra implies that the collection of assignments

$$(H_1, H_2) \rightarrow \exp[(H_1 + H_2, H_C)]$$

is a two-variable Detroit family, call it again $e^{\mathbf{H}}$.

§3. c-Systems

Let (V, Φ) be a geometric g.r.s. It will be technically convenient not to assume that Φ spans.

Motivated as always by the theory from [1-(b)], we are led to the following definition.

Definition. A c-system on (V, Φ) consists in the assignment to each chamber C of a finite dimensional Hilbert space $\mathcal{H}_C$ and the assignment to each pair of chambers C', C'' of a C^∞ function

$$\mathbf{c}(C'', C' :?) : \sqrt{-1}V \to \text{End } (\mathcal{H}_{C'}, \mathcal{H}_{C''})$$

that is holomorphic in a witch region, subject to

$$(c_1) \quad \mathbf{c}(C''', C'' :?) \bullet \mathbf{c}(C'', C' :?) = \mathbf{c}(C''', C' :?)$$

$$(c_2) \quad \mathbf{c}(C, C :?) = \text{ID}_{\mathcal{H}_C}$$

$$(c_3) \quad \mathbf{c}(C'', C' :?)^* = \mathbf{c}(C', C'' : -\overline{?})$$

$$(c_4) \quad \mathbf{c}(C'', C' : H') = \mathbf{c}(C'', C' : H'')$$

if

$$(\lambda, H') = (\lambda, H'')$$

when C' and C'' are adjacent with common wall in V_λ.

Remark. From the Detroit family viewpoint, the last condition is important. Thus fix a chamber C_0 and put

$$\Gamma_{\mathbf{c},C}(C_0 : H_1, H_2) = \mathbf{c}(C_0, C : -H_2) \bullet \mathbf{c}(C, C_0 : H_1).$$

Claim:

$$\{\Gamma_{\mathbf{c},C}(C_0 :?)\}$$

is a two-variable Detroit family (of operators). To make the verification, we shall use Theorem 2.1. Let, therefore, C' and C'' be adjacent with common wall in V_λ – then

$$(H_1, H_2) \perp (\lambda, \lambda) \Rightarrow (H_1, \lambda) = (-H_2, \lambda)$$

and so

$$\begin{aligned}\Gamma_{\mathbf{c},C''}(C_0 : H_1, H_2) &= \mathbf{c}(C_0, C'' : -H_2) \bullet \mathbf{c}(C'', C_0 : H_1)\\ &= \mathbf{c}(C_0, C' : -H_2) \bullet [\mathbf{c}(C', C'' : -H_2)\\ &\qquad \bullet \mathbf{c}(C'', C' : H_1)] \bullet \mathbf{c}(C', C_0 : H_1)\\ &= \mathbf{c}(C_0, C' : -H_2) \bullet \mathbf{c}(C', C_0 : H_1)\\ &= \Gamma_{\mathbf{c},C'}(C_0 : H_1, H_2).\end{aligned}$$

Hence the claim.

Relative to a given c-system on (V, Φ), a c-compatible family of operators is, by definition, the assignment to each chamber C of a C^∞ function

$$f(C:?) : \sqrt{-1}V \to \text{End}\,(\mathcal{H}_C)$$

that is holomorphic in a witch region such that $\forall C', C''$:

$$f(C'':?) \bullet \mathbf{c}(C'', C':?) = \mathbf{c}(C'', C':?) \bullet f(C':?).$$

By an automorphism of the c-system and its associated c-compatible family of operators, we mean an orthogonal transformation

$$\sigma : V \to V$$

that preserves Φ, with $\langle \sigma \rangle$ operating simply on the chambers, such that

$$\text{Im}(1-\sigma)$$

is special, subject to:

(σ_1) $\mathcal{H}_{\sigma C} = \mathcal{H}_C$

(σ_2) $\mathbf{c}(\sigma C'', \sigma C' : \sigma ?) = \mathbf{c}(C'', C':?)$

(σ_3) $f(\sigma C : \sigma ?) = f(C:?)$.

Let

$$\text{Vec}_\Phi$$

be the vector space of Φ-orthogonal sets $\mathbf{H} = \{H_C\}$. For each C, define

$$T_C : \text{Vec}_\Phi \to V$$

by the requirement

$$T_C(\mathbf{H}) = H_C.$$

Let

$$\text{Vec}_\Phi^\sigma = \{\mathbf{H} \in \text{Vec}_\Phi : \forall C, H_{\sigma C} = \sigma H_C\}.$$

We shall then assume in addition that

$$\begin{cases} \forall C, T_C \mid \text{Vec}_\Phi^\sigma \text{ is onto} \\ \exists \mathbf{H} \in \text{Vec}_\Phi^\sigma \text{ such that } \forall C,\ H_C \in C. \end{cases}$$

§4. A Conjecture

Let there be given a **c**-system on (V, Φ), as well as a **c**-compatible family of operators. To ensure the existence of the integrals infra, we shall assume that

$$\mathbf{c}(C'', C' :?)$$

and its derivatives are slowly increasing on $\sqrt{-1}V$, while

$$f(C :?)$$

and its derivatives are rapidly decreasing on $\sqrt{-1}V$.

Fix an automorphism σ. Put

$$C_0 - \phi_C^{\sigma}(H_1, H_2) = \mathrm{tr}(\Gamma_{\mathbf{c},C}(C_0 : H_1, H_2) \bullet \mathbf{c}(C_0 : \sigma^{-1}C_0 : H_1) \bullet f(\sigma^{-1}C_0 : H_1)).$$

Then

$$C_0 - \phi^{\sigma}$$

is a two-variable Detroit family, as is the product

$$e^{\mathbf{H}} \bullet C_0 - \phi^{\sigma} \qquad (\mathbf{H} \in \mathrm{Vec}_{\Phi}^{\sigma}).$$

Thanks to Theorem 2.1, it thus makes sense to consider

$$\mathrm{III}_{e^{\mathbf{H}} \bullet C_0 - \phi^{\sigma}}$$

or, more generally,

$$\mathrm{III}_{e^{\mathbf{H}} \bullet C_0 - \phi^{\sigma}(C_W)},$$

W a special subspace of V, the ambient g.r.s. then being $\Phi(W)$ rather than Φ (but taken in all of V).

On the basis of what has been said in [1-(b), §9], it is natural to introduce the following function of

$$\mathbf{H}, \mathbf{H}_O \in \mathrm{Vec}_{\Phi}^{\sigma},$$

namely

$$\Omega_{\sigma}(\mathbf{H}, \mathbf{H}_O) = \sum_{W : W \supset \mathrm{Im}(1-\sigma)} \sum_{C_W} \frac{1}{C(W)} \sum_{C(W)}$$

$$p(F_0(C_W) : H_C - H_{O,C}) \bullet \int_{\sqrt{-1}V} \mathrm{III}_{(e^{\mathbf{H}} \bullet C - \phi^{\sigma})(C_W)}(H, -\sigma H) |dH|.$$

Here, of course, W is special, with

$$C \leftrightarrow (C_W, C(W)),$$

and p is the Arthur polynomial.

Conjecture. $\forall \mathbf{H}_O$,

$$\Omega_\sigma(\mathbf{H}, \mathbf{H}_O)$$

is bounded as a function of $\mathbf{H}$.

We refer the reader to [1-(c), §4] for the background and relevance of this statement.

§5. Descent

Suppose that W is a special subspace of V. Fix a chamber C_W of $W^\perp$ – then, starting from our given c-system in (V, Φ), we can produce a c-system on $(W, \Phi(W))$ as follows. Put

$$\mathcal{H}_{C(W)} = \mathcal{H}_C \qquad (C \leftrightarrow (C_W, C(W)))$$

and let

$$\mathbf{c}_{C_W}(C''(W), C'(W) :?) = \mathbf{c}(C'', C' :?)$$

if

$$\begin{cases} C' \leftrightarrow (C_W, C'(W)) \\ C'' \leftrightarrow (C_W, C''(W)). \end{cases}$$

It is easy to verify that the axioms (c_1) – (c_4) of §3 are satisfied.

To get an associated $\mathbf{c}_{C_W}$-compatible family of operators from the one at hand, let

$$f_{C_W}(C(W) : H) = \int_{\sqrt{-1}W^\perp} f(C : H + H^\perp)|dH^\perp|.$$

Each such is certainly Schwartz on $\sqrt{-1}W$. And,

$$f_{C_W}(C''(W) : H) \bullet \mathbf{c}_{C_W}(C''(W), C'(W) : H)$$

$$\begin{aligned}
&= \int_{\sqrt{-1}W^\perp} f(C'' : H + H^\perp) \bullet \mathbf{c}(C'', C' : H)|dH^\perp| \\
&= \int_{\sqrt{-1}W^\perp} f(C'' : H + H^\perp) \bullet \mathbf{c}(C'', C' : H + H^\perp)|dH^\perp| \\
&= \int_{\sqrt{-1}W^\perp} \mathbf{c}(C'', C' : H + H^\perp) \bullet f(C' : H + H^\perp)|dH^\perp| \\
&= \int_{\sqrt{-1}W^\perp} \mathbf{c}(C'', C' : H) \bullet f(C' : H + H^\perp)|dH^\perp| \\
&= \mathbf{c}_{C_W}(C''(W), C'(W) : H) \bullet f_{C_W}(C'(W) : H).
\end{aligned}$$

[Note: We have tacitly used the fact that

$$\mathbf{c}(C'', C' : H + H^\perp)$$

is independent of $H^\perp$, as can be readily seen when $C'(W)$ and $C''(W)$ are at first adjacent, then in general.]

In terms of the automorphism σ, the W of interest are those that contain $\mathrm{Im}(1-\sigma)$. If W has this property, then W is σ-invariant, so σ induces an automorphism of $\Phi(W)$. Furthermore, since σ is the identity on $W^{\perp} \subset \mathrm{Ker}(1-\sigma)$,

$$C \leftrightarrow (C_W, C(W)) \Rightarrow \sigma C \leftrightarrow (C_W, \sigma C(W)).$$

Consequently, σ is an automorphism of

$$\begin{cases} \mathbf{c}_{C_W} \\ f_{C_W}. \end{cases}$$

Assigning to the symbols

$$\begin{cases} \mathrm{Vec}_{\Phi(W)} \\ \mathrm{Vec}_{\Phi(W)}^{\sigma} \end{cases}$$

the evident meanings, there are canonical maps

$$\begin{cases} \mathrm{Vec}_{\Phi} \rightarrow \mathrm{Vec}_{\Phi(W)} \\ \mathrm{Vec}_{\Phi}^{\sigma} \rightarrow \mathrm{Vec}_{\Phi(W)}^{\sigma}, \end{cases}$$

viz.

$$\mathbf{H} \rightarrow \mathbf{H}_{C_W},$$

where

$$\mathbf{H}_{C_W}(C(W)) = \mathrm{Pro}_W \mathbf{H}(C) \qquad (C \leftrightarrow (C_W, C(W))).$$

§6. The Theorem of Reduction

We shall now examine the function

$$\Omega_\sigma(\mathbf{H}, \mathbf{H}_O)$$

appearing in the conjecture of §4 somewhat more closely.

Fix a special

$$W \supset \mathrm{Im}(1-\sigma).$$

Then

$$\int_{\sqrt{-1}V} Ш_{(e^{\mathbf{H}}\bullet C-\phi^\sigma)(C_W)}(H, -\sigma H)|dH|$$

$$= \int_{\sqrt{-1}W}\int_{\sqrt{-1}W^\perp} Ш_{(e^{\mathbf{H}}\bullet C-\phi^\sigma)(C_W)}(H+H^\perp, -\sigma H - \sigma H^\perp)|dH^\perp|\ |dH|$$

$$= \int_{\sqrt{-1}W}\int_{\sqrt{-1}W^\perp} Ш_{(e^{\mathbf{H}}\bullet C-\phi^\sigma)(C_W)}(H+H^\perp, -\sigma H - H^\perp)|dH^\perp|\ |dH|$$

σ being the identity on $W^\perp$.

For H', H'' in general position, the definitions imply that

$$\int_{\sqrt{-1}W^\perp} Ш_{(e^{\mathbf{H}}\bullet C-\phi^\sigma)(C_W)}(H'+H^\perp, H''-H^\perp)|dH^\perp|$$

$$= Ш_{(e^{\mathbf{H}}C_W\bullet C(W)-\phi^\sigma)}(H', H'').$$

Here,

$$C(W) - \phi^\sigma$$

is manufactured from $\mathbf{c}_{C_W}$ and f_{C_W} just as

$$C - \phi^\sigma$$

was manufactured from $\mathbf{c}$ and f.

Accordingly,

$$\int_{\sqrt{-1}V} Ш_{(e^{\mathbf{H}}\bullet C-\phi^\sigma)(C_W)}(H, -\sigma H)|dH|$$

$$= \int_{\sqrt{-1}W} Ш_{(e^{\mathbf{H}}C_W\bullet C(W)-\phi^\sigma)}(H, -\sigma H)|dH|.$$

Agreeing to write

$$C_W - \phi^\sigma$$

in place of

$$\frac{1}{C(W)} \bullet \sum_{C(W)} C(W) - \phi^\sigma,$$

we then find that

$$\Omega_\sigma(\mathbf{H}, \mathbf{H}_O) = \sum_{W:W\supset \mathrm{Im}(1-\sigma)} \sum_{C_W} p(F_0(C_W) : H(C_W) - H_O(C_W))$$

$$\bullet \int_{\sqrt{-1}W} \mathrm{III}_{(e^{\mathbf{H}C_W} \bullet C(W) - \phi^\sigma)}(H, -\sigma H)|dH|.$$

Needless to say,

$$\begin{cases} H(C_W) = \mathrm{Pro}_{W^\perp} H_C \\ H_O(C_W) = \mathrm{Pro}_{W^\perp} H_{O,C}, \end{cases}$$

as is permissible.

To go further, it will be necessary to use the addition rule for Arthur polynomials (cf. [1-(b), §3]). Thus

$$\Omega_\sigma(\mathbf{H}, \mathbf{H}_O) = \sum_{W:W\supset \mathrm{Im}(1-\sigma)} \sum_{C_W} p(F_0(C_W) : H'(C_W) - H_O(C_W) + H(C_W) - H'(C_W))$$

$$\bullet \int_{\sqrt{-1}W} \mathrm{III}_{(e^{\mathbf{H}C_W} \bullet C(W) - \phi^\sigma)}(H, -\sigma H)|dH|$$

$$= \sum_{W:W\supset \mathrm{Im}(1-\sigma)} \sum_{C_W} \sum_{F\subset F_0(C_W)} p(F_0(C_W) - F : H'(C_W) - H_O(C_W))$$

$$\bullet p(F : H(C_W) - H'(C_W))$$

$$\bullet \int_{\sqrt{-1}W} \mathrm{III}_{(e^{\mathbf{H}C_W} \bullet C(W) - \phi^\sigma)}(H, -\sigma H)|dH|.$$

Put

$$W_0 = W + W^\perp(F).$$

Then W_0 is special and $W_0 \supset W$. The pairs F, C_W for which $F \subset F_0(C_W)$ correspond to quadruples

$$(W, W_0, C_{W_0}, C_W^\dagger),$$

where C_{W_0} is the chamber arising from the $F_0(C_W) - F$ projection to $W_0^\perp$ and $C_W^\dagger$ is the chamber in $W_0 \cap W^\perp$ picked off by F. All such quadruples arise bijectively. So we get

$$\Omega_\sigma(\mathbf{H}, \mathbf{H}_O) = \sum_{W_0, C_{W_0}} p(F_0(C_W) : H'(C_{W_0}) - H_O(C_W))$$

$$\bullet \sum_{W, C_W^\dagger} p(F_0(C_W^\dagger) : H(C_W) - H'(C_W))$$

$$\bullet \int_{\sqrt{-1}W} Ш_{(e^{\mathbf{H}_{C_W}} \bullet C(W) - \phi^\sigma)}(H, -\sigma H)|dH|$$

$$= \sum_{W_0, C_{W_0}} p(F_0(C_{W_0}) : H'(C_{W_0}) - H_{\mathrm{O}}(C_{W_0})) \bullet \Omega_\sigma(\mathbf{H}_{C_{W_0}}, \mathbf{H}'_{C_{W_0}}).$$

This step is justified by the observation that descent is transitive, i.e., the triangle

$$\begin{array}{ccc} V & \xrightarrow{\text{DES}} & W_0 \\ & {\scriptstyle \text{DES}} \searrow & \downarrow {\scriptstyle \text{DES}} \\ & & W \end{array}$$

is commutative.

Dropping the subscript from the notation then leads to the following theorem.

Theorem 6.1. $\forall \mathbf{H}, \mathbf{H}_{\mathrm{O}} \in \mathrm{Vec}_{\Phi}^{\sigma}$,

$$\Omega_\sigma(\mathbf{H}, \mathbf{H}_{\mathrm{O}}) = \sum_{W, C_W} p(F_0(C_W) : H'(C_W) - H_{\mathrm{O}}(C_W)) \bullet \Omega_\sigma(\mathbf{H}_{C_W}, \mathbf{H}'_{C_W}).$$

It seems likely that this result may be helpful in an attempt to establish the conjecture formulated earlier. We shall close with a remark in this direction.

Claim: If the conjecture is true for $\mathbf{H}_{\mathrm{O}} = 0$, then the conjecture is true for *all* $\mathbf{H}_{\mathrm{O}}$. In fact, taking $\mathbf{H}' = 0$, we can assume inductively that the various

$$\Omega_\sigma(\mathbf{H}_{C_W}, ?)$$

are bounded unless $W = V$ – then

$$\Omega_\sigma(\mathbf{H}, \mathbf{H}_{\mathrm{O}}) = \Omega_\sigma(\mathbf{H}, 0) + \text{ a bounded function of } \mathbf{H}.$$

Hence the claim.

The conjecture is therefore "base-point" invariant.

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