Math 307- Summer 2004

Basic Formulas

Algebra

Completing the square: $X^2 + bX + c = (X + \frac{b}{2})^2 \; - \frac{b^2}{4} + c\,.$

Quadratic formula: roots of $aX^2 + bX + c$ are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}\,.$

Exponents: $a^b \cdot a^c = a^{b+c}; \quad \dfrac{a^b}{a^c} = a^{b-c}; \quad (a^b)^c = a^{bc}; \quad a^{1/b} = \sqrt[b]{a}\,.$

Geometry

Circle: circumference $= 2\pi r$; area $= \pi r^2$. Sphere: vol $= \frac{4}{3}\pi r^3$; surface area $= 4\pi r^2$.

Cylinder: vol $= \pi r^2 h$; lateral area $= 2\pi rh$; total surface area $= 2\pi rh + 2\pi r^2$.

Cone: vol $= \frac{1}{3}\pi r^2 h$; lateral area $= \pi r\sqrt{r^2 + h^2}$; total surface area $= \pi r\sqrt{r^2 + h^2} + \pi r^2$.

Analytic geometry

Point-slope formula for straight line: $y = y_0 + m(x - x_0)$.

Circle centered at (h, k): $(x - h)^2 + (y - k)^2 = r^2$.

Ellipse with semimajor axis along x-axis and semiminor axis along y-axis: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

Trigonometry

$\sin = \dfrac{\text{opposite}}{\text{hypotenuse}}; \ \cos = \dfrac{\text{adjacent}}{\text{hypotenuse}}; \ \tan = \dfrac{\text{opposite}}{\text{adjacent}};$

$\sec = \dfrac{1}{\cos}; \ \csc = \dfrac{1}{\sin}; \ \cot = \dfrac{1}{\tan}; \ \tan = \dfrac{\sin}{\cos}; \ \cot = \dfrac{\cos}{\sin};$

$\sin x = \cos\left(\dfrac{\pi}{2} - x\right); \ \cos x = \sin\left(\dfrac{\pi}{2} - x\right);$

$\sin(x + \pi) = -\sin x; \ \cos(x + \pi) = -\cos x.$

Law of sines: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$. Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos A$.

Sum of angles: $\sin(x + y) = \sin x\cos y + \cos x\sin y$; $\cos(x + y) = \cos x\cos y - \sin x\sin y$;

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x\,\tan y}$$

$\sin^2$ and $\cos^2$ formulas: $\sin^2 x + \cos^2 x = 1$; $\tan^2 x + 1 = \sec^2 x$; $1 + \cot^2 x = \csc^2 x$;

$$\sin^2 x = \frac{1 - \cos(2x)}{2}; \ \cos^2 x = \frac{1 + \cos(2x)}{2}.$$

Product formula: $\cos x - \cos y = 2\,\sin\left(\dfrac{x + y}{2}\right)\,\sin\left(\dfrac{y - x}{2}\right)$.

Calculus.

Basic differentiation formulae: $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$, $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$.

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v^2}\left(v\frac{du}{dx} - u\frac{dv}{dx}\right), \text{ for } v \neq 0.$$

Chain rule: $\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$.

Fundamental Theorem of Calculus:

if $F'(x) = f(x)$ then $\int_a^b f(x)dx = F(b) - F(a)$ and $\int f(x)dx = F(x) + C$

Differentiation of limits of integral: $\frac{d}{dx}\int_{f(x)}^{g(x)} h(u)\,du = h\left(g(x)\right)g'(x) - h\left(f(x)\right)f'(x)$

Derivatives of specific functions: $\frac{dx^n}{dx} = nx^{n-1}$; $\frac{de^x}{dx} = e^x$; $\frac{d\ln|x|}{dx} = \frac{1}{x}$;

$\frac{d\sin x}{dx} = \cos x$; $\frac{d\cos x}{dx} = -\sin x$; $\frac{d\tan x}{dx} = \sec^2 x$;

$\frac{d\text{Arcsin}\,x}{dx} = \frac{1}{\sqrt{1-x^2}}$; $\frac{d\text{Arctan}\,x}{dx} = \frac{1}{1+x^2}$.

Basic integration formulae: $\int (u+v)\,dx = \int u\,dx + \int v\,dx$; $\int au\,dx = a\int u\,dx$;

Substitution: $\int f\left(u(x)\right)u'(x)\,dx = F\left(u(x)\right)$, where $\int f(u)du = F(u)$;

Integration by parts: $\int u\,dv = uv - \int v\,du$;

Standard integrals:

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C\ (n \neq -1)$; $\int \frac{dx}{x} = \ln|x| + C$; $\int e^x\,dx = e^x + C$;

$\int \sin x\,dx = -\cos x + C$; $\int \cos x\,dx = \sin x + C$; $\int \tan x\,dx = -\ln|\cos x| + C$;

$\int \frac{dx}{\sqrt{1-x^2}} = \text{Arcsin}\,x + C$; $\int \frac{xdx}{\sqrt{1-x^2}} = -\sqrt{1-x^2} + C$;

$\int \frac{dx}{1+x^2} = \text{Arctan}\,x + C$; $\int \frac{x\,dx}{1+x^2} = \frac{1}{2}\ln(1+x^2) + C$

Rational with quadratic denominator:

If the degree of the numerator is not smaller than the degree of the denominator, divide.

If the denominator does not factor use above two integrals and substitution.

If the denominator does factor, then write

$\frac{ax+b}{(x-c)(x-d)} = \frac{A}{x-c} + \frac{B}{x-d}$ which is the same as: $ax+b = A(x-d) + B(x-c)$

Set x=c then x=d to find A and B, given a and b.